Warm-up

1) $y = 6(x+7)^2 - 1$

Transformation:

Vert stretch 6 L 7 D 1

Range: $[-1, \infty)$

Increasing: $(-7, \infty)$

End Behavior:

$x \to -\infty \quad y \to \infty$

$x \to \infty \quad y \to \infty$

Domain:

$(-\infty, \infty)$

Decreasing:

$(-\infty, -7)$

2) $y = 2\sqrt{x-6} + 3$

Transformation:

Vert stretch 2 R 6 U 3

Range:

$[3, \infty)$

Increasing: $[3, \infty)$

End Behavior:

$x \to 6 \quad y \to 3$

$x \to \infty \quad y \to \infty$

Domain:

$[6, \infty)$

Decreasing: N/A

Homework

1. T: Left 5 D: $[-5, \infty)$ R: $(0, \infty)$
   EB: As $x \to \infty, y \to \infty$.
   Increasing: $(-5, \infty)$ Decreasing: None

2. T: Down 3 D: $(-\infty, \infty)$ R: $(-\infty, \infty)$
   EB: As $x \to \infty, y \to \infty$. ; As $x \to -\infty, y \to -\infty$.
   Increasing: $(-\infty, \infty)$ Decreasing: None

Homework

3. T: Left 5, Vertical stretch by 4

   D: $(-5, \infty)$ R: $(0, \infty)$

   EB: As $x \to \infty, y \to \infty$.

   Increasing: $(-5, \infty)$ Decreasing: None

4. T: Reflect across x-axis; Vertical Stretch by 2; Right 1; Down 4

   D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

   EB: As $x \to \infty, y \to \infty$. ; As $x \to -\infty, y \to -\infty$.

   Increasing: None Decreasing : $(-\infty, \infty)$

Homework

5. T: Vertical shrink by $\frac{1}{5}$; Left 4

   D: $[-4, \infty)$ R: $(0, \infty)$

   EB: As $x \to \infty, y \to \infty$.

   Increasing: $(-4, \infty)$ Decreasing: None

6. T: Reflect across x-axis; Vertical Stretch by 5; Left 5; Down 7

   D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

   EB: As $x \to \infty, y \to \infty$. ; As $x \to -\infty, y \to -\infty$.

   Increasing: None Decreasing : $(-\infty, \infty)$

Homework

7. T: Reflect across x-axis; Vertical shrink by $\frac{1}{4}$; Right 6; Up 2

   D: $[6, \infty)$ R: $(-\infty, 2]$

   EB: As $x \to \infty, y \to -\infty$.

   Increasing: None Decreasing : $(6, \infty)$

8. T: Left 4; Up 6

   D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

   EB: As $x \to \infty, y \to \infty$. ; As $x \to -\infty, y \to -\infty$.

   Increasing: $(-\infty, \infty)$ Decreasing: None

Homework

9. T: Vertical shrink by $\frac{2}{5}$; Right 1; Up 3

   D: $[1, \infty)$ R: $(3, \infty)$

   EB: As $x \to \infty, y \to \infty$.

   Increasing: $(-4, \infty)$ Decreasing: None

Graphing Reciprocal Functions

None of this is
Inverse variation

Graphing Reciprocal Functions

- Consider the function $y = \frac{5}{x}$
- Without graphing, can you find any limitations for x?

The denominator of a fraction can never equal 0, so $x \neq 0$.

- How about limitations for y?
- Since $xy = 5$, $y \neq 0$

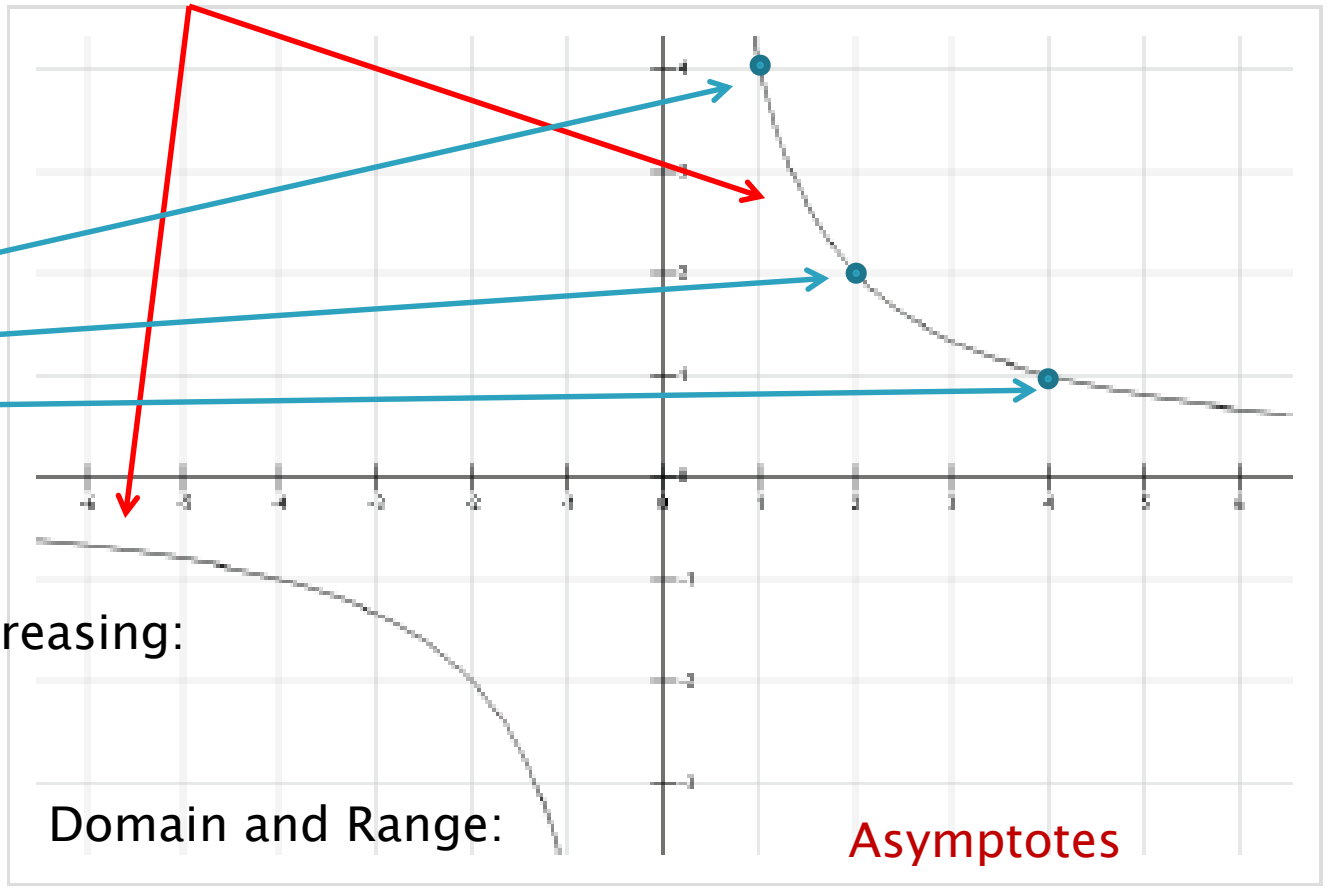
The graph and its parts

$$y = \frac{4}{x}$$

x	y
1	4
2	2
4	1

Branches

End Behavior:



Increasing/Decreasing:

Domain and Range:

Asymptotes

x=0, y=0

Transformation of inverse variations

Reciprocals

- Given the parent function $y = \frac{4}{x}$
- What do you think the following transformations will do to the function?
- 1. $y = \frac{4}{x+2}$ Left 2 2. $y = \frac{4}{x} + 7$ up 7 3. $y = \frac{-4}{x}$ Reflection x-axis
- Check your answer in your calculator.
  (Were you right?)
- What do you notice about the asymptotes for the transformed functions?

$x = -2$ $y = 0$

$x = 0$ $y = 7$

$x = 0$ $y = 0$

The Properties

- Translations of Inverse Variations

Also Vert stretch shrink

The graph of $y = \frac{k}{x-b} + c$ is a translation of $y = \frac{k}{x}$ by *b* units horizontally and *c* units vertically. The vertical asymptote is $x=b$. The horizontal asymptote is $y = c$.

When k is positive:

the graph is in quadrants I and III

When k is negative:

the graph is in quadrants II and IV (reflection)

Describe the transformations, name the asymptotes, domain and range

$$A. y = \frac{3}{x+4} + 1$$

Left 4 Up 1

$X = -4$ $Y = 1$

D $(-\infty, -4) \cup (-4, \infty)$

R $(-\infty\ 1) \cup (1, \infty)$

$$B. y = \frac{-2}{x} + 5$$

Up 5 Reflect x-axis

$X = 0$ $Y = 5$

D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty\ , 5) \cup (5, \infty)$

$$C. y = \frac{5}{x+1} - 3$$

Left 1 Down 3

$X = -1$ $Y = 3$

D: $(-\infty, -1) \cup (-1, \infty)$

R: $(-\infty, 3) \cup (3, \infty)$

Try some on your own:

$$D. y = \frac{-1}{x+3} + 2$$

L 3 U 2

Reflect x axis

$X = -3$ $Y = 2$

D: $(-\infty, -3) \cup (-3, \infty)$

R: $(-\infty, 2) \cup (2, \infty)$

$$E. y = \frac{2}{x+1} - 1$$

L 1 D 1

$X = -1$ $Y = -1$

D: $(-\infty, -1) \cup (-1, \infty)$

R: $(-\infty, -1) \cup (-1, \infty)$

$$F. y = \frac{4}{x - 5} + 4$$

$x + 5$

L 5 U 4

$X = -5$ $Y = 4$

D: $(-\infty, -5) \cup (-5, \infty)$

R: $(-\infty, 4) \cup (4, \infty)$

Examples

1. Write the equation if the parent function $y = \frac{2}{x}$ has asymptotes at $x = -2$ and $y = 3$.

Left 2 Up 3

$y = \frac{2}{x+2} + 3$

- 2. For the same parent function, write the equation for the translation of that is 4 units left and 5 units up

Left 4 $x = -4$

+4 (in) $x+4$

$y = 5$ Up 5

+5 (out)

$y = \frac{2}{x+4} + 5$

Examples

1. Write the equation if the parent function $y = \frac{3}{x}$ has asymptotes at $x = -1$ and $y = 5$.

$y - 5 = \frac{3}{x+1} \rightarrow y = \frac{3}{x+1} + 5$

$x+1$ $y-5$

- 2. For the same parent function, write the equation for the translation of that is 7 units right and 2 units down.

$x-7$ in

-2 out

$y = \frac{3}{x-7} - 2$

Examples

1. Write the equation if the parent function $y = \frac{5}{x}$ has asymptotes at $x = 8$ and $y = -4$.

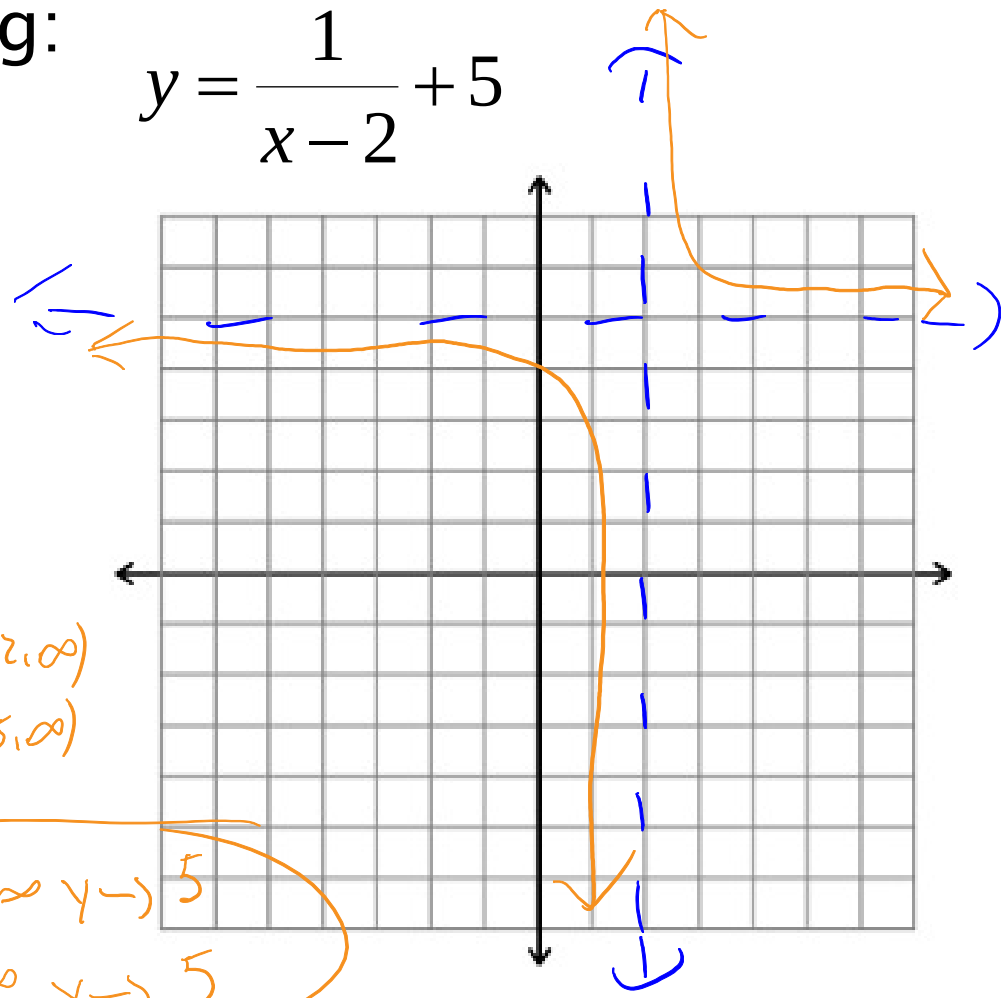
$y+4 = \frac{5}{x-8} \rightarrow y = \frac{5}{x-8} - 4$

$x-8$ $y+4$

- 2. For the same parent function, write the equation for the translation of that is 7 units up and 2 units left.

$x+2$ (in) $y = \frac{5}{x+2} + 7$ $+7$ out

- Graph the following: $y = \frac{1}{x-2} + 5$



- Find the:

a. Asymptotes

$x = 2$ $y = 5$

b. Domain and Range

D: $(-\infty, 2) \cup (2, \infty)$

R: $(-\infty, 5) \cup (5, \infty)$

c. End Behavior $x \to -\infty$ $y \to 5$

$x \to \infty$ $y \to 5$

d. Intervals of increasing and decreasing

N/A $(-\infty, 2) \cup (2, \infty)$

- Graph the following: $y = \frac{2}{x+1} - 2$

- Find the:

a. Asymptotes

$x = -1 \quad y = -2$

b. Domain and Range

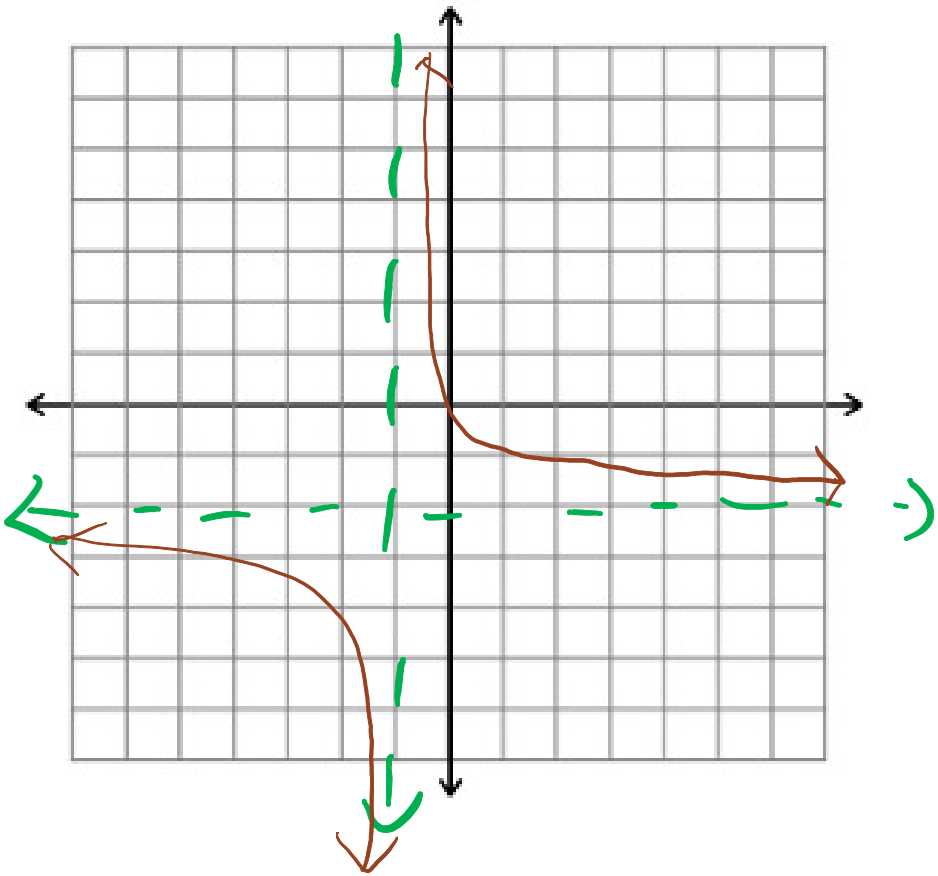
D: $(-\infty, -1) \cup (-1, \infty)$

R: $(-\infty, -2) \cup (-2, \infty)$

c. End Behavior

$x \to -\infty \quad y \to -2$

$x \to \infty \quad y \to -2$



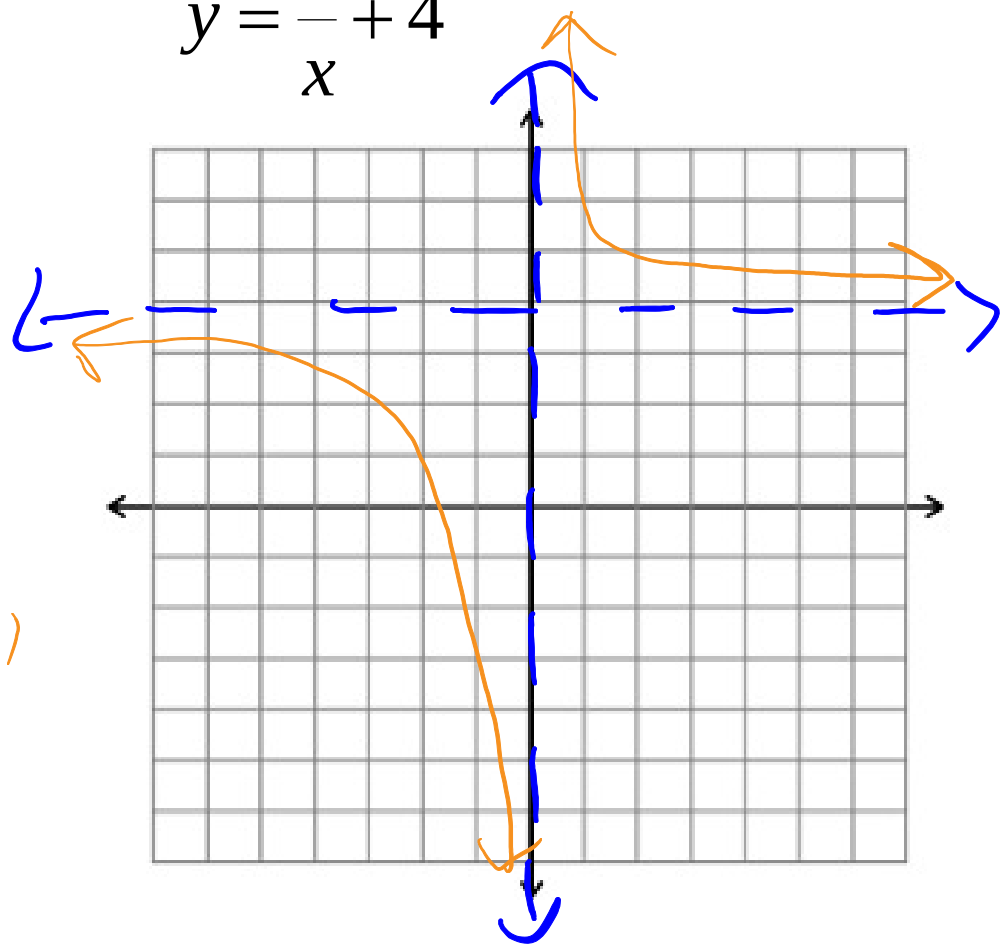
d. Intervals of increasing and decreasing

N/A

$(-\infty, -1) \cup (-1, \infty)$

- Graph the following:

$$y = \frac{2}{x} + 4$$



- Find the:

a. Asymptotes

$x=0$ $y=4$

b. Domain and Range

D: $(-\infty,0)\cup(0,\infty)$

R: $(-\infty,4)\cup(4,\infty)$

c. End Behavior

$x \to -\infty$ $y \to 4$

$x \to \infty$ $y \to 4$

d. Intervals of increasing and decreasing

N/A

$(-\infty,0)\cup(0,\infty)$

- Graph the following: $y = \frac{2}{x+3} - 4$

- Find the:

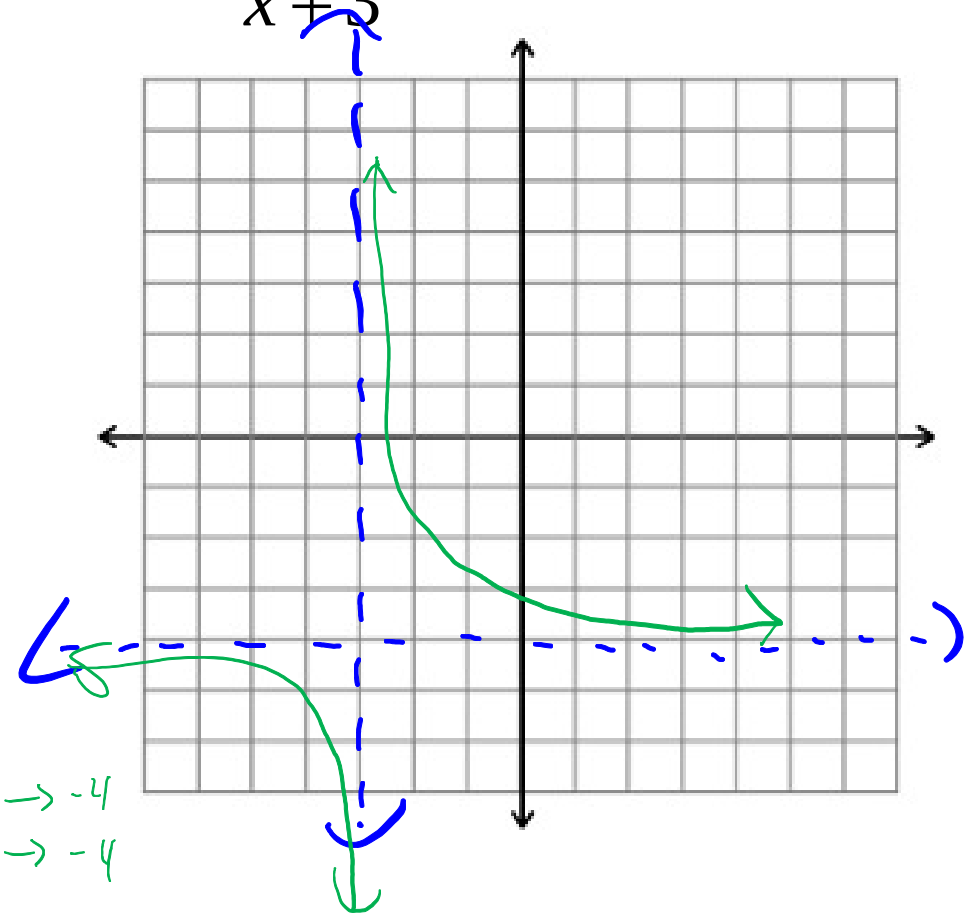
a. Asymptotes

$x = -3 \quad y = -4$

b. Domain and Range

D: $(-\infty, -3) \cup (-3, \infty)$

R: $(-\infty, -4) \cup (-4, \infty)$



c. End Behavior

$x \to -\infty \quad y \to -4$

$x \to \infty \quad y \to -4$

d. Intervals of increasing and decreasing

N/A

$(-\infty, -3) \cup (-3, \infty)$

Example:

Skip

Reciprocal

- Write and graph the ~~inverse variation~~ in which $y = \frac{1}{2}$ and $x = 10$.

What are/is the:

Asymptotes?

End Behavior?

Domain and Range?

Interval notation?