FACTORING OUT GCF

1) COMPLETE A FACTOR TREE FOR 84

$$
\begin{aligned}
84 &= \\
&= 4 \cdot 21 \\
&= 2 \cdot 2 \cdot 3 \cdot 7 \\
&= \mathbf{2^2 \cdot 3 \cdot 7}
\end{aligned}
$$

2) Complete a factor tree for -210

$$
\begin{aligned}
-210 &= \\
&= -1 \cdot 210 \\
&= -1 \cdot 30 \cdot 7 \\
&= -1 \cdot 6 \cdot 5 \cdot 7 \\
&= \mathbf{-1 \cdot 2 \cdot 3 \cdot 5 \cdot 7}
\end{aligned}
$$

3) FIND THE PRIME FACTORIZATION OF $45A^2B^3$ (SAME AS A FACTOR TREE)

$$45a^2b^3 = 9 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot b$$

$$= 3 \cdot 3 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot b$$

$$= \mathbf{3^2 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot b}$$

Write the variables without exponents.

WHAT IS THE PRIME FACTORIZATION OF 48?

1. $3 \cdot 16$
2. $3 \cdot 4 \cdot 4$
3. $2 \cdot 2 \cdot 3 \cdot 4$
4. ✓ $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

THE **GREATEST COMMON FACTOR (GCF)** OF 2 OR MORE NUMBERS IS

the largest number that can divide into all of the numbers.

4) Find the GCF of 42 and 60.

Write the prime factorization of each number.

4) FIND THE GCF OF 42 AND 60.

$$42 = 2 \cdot 3 \cdot 7$$
$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

What prime factors do the numbers have in common?

Multiply those numbers.

The GCF is $2 \cdot 3 = 6$

6 is the largest number that can go into 42 and 60!

5) FIND THE GCF OF $40A^2B$ AND $48AB^4$.

$40a^2b = 2 \cdot 2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot b$

$48ab^4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot b \cdot b \cdot b \cdot b$

What do they have in common?

Multiply the factors together.

GCF = **8ab**

WHAT IS THE GCF OF 48 AND 64?

1. 2
2. 4
3. 8
4. 16 ✓

REVIEW: WHAT IS THE GCF OF $25A^2$ AND 15A?

5a

Let's go one step further…

1) **FACTOR** $25a^2 + 15a$.

Find the GCF and divide each term

$$25a^2 + 15a = 5a(\underline{5a} + \underline{3})$$

Check your answer by distributing.

2) FACTOR $18X^2 - 12X^3$.

Find the GCF

$6x^2$

Divide each term by the GCF

$18x^2 - 12x^3 = 6x^2(\underline{\ 3\ } - \underline{\ 2x\ })$

Check your answer by distributing.

3) FACTOR $28A^2B + 56ABC^2$.

GCF = 28ab

Divide each term by the GCF

$28a^2b + 56abc^2 = 28ab\left(\frac{a}{\uparrow} + \frac{2c^2}{\uparrow}\right)$

Check your answer by distributing.

$28ab(a + 2c^2)$

4) FACTOR $20X^2 - 24XY$

1. $x(20 - 24y)$
2. $2x(10x - 12y)$
3. $4(5x^2 - 6xy)$
4. ✓ $4x(5x - 6y)$

5) FACTOR $28A^2 + 21B - 35B^2C^2$

GCF = 7

Divide each term by the GCF

$$28a^2 + 21b - 35b^2c^2 = 7\left(\frac{4a^2}{\uparrow} + \frac{3b}{\uparrow} - \frac{5b^2c^2}{\uparrow}\right)$$

Check your answer by distributing.

$$7(4a^2 + 3b - 5b^2c^2)$$

FACTORING GCF

- Always factor out the GCF before attempting any other method of factoring.
- Work on homework

WARM UP DAY 2 FACTOR BY GROUPING

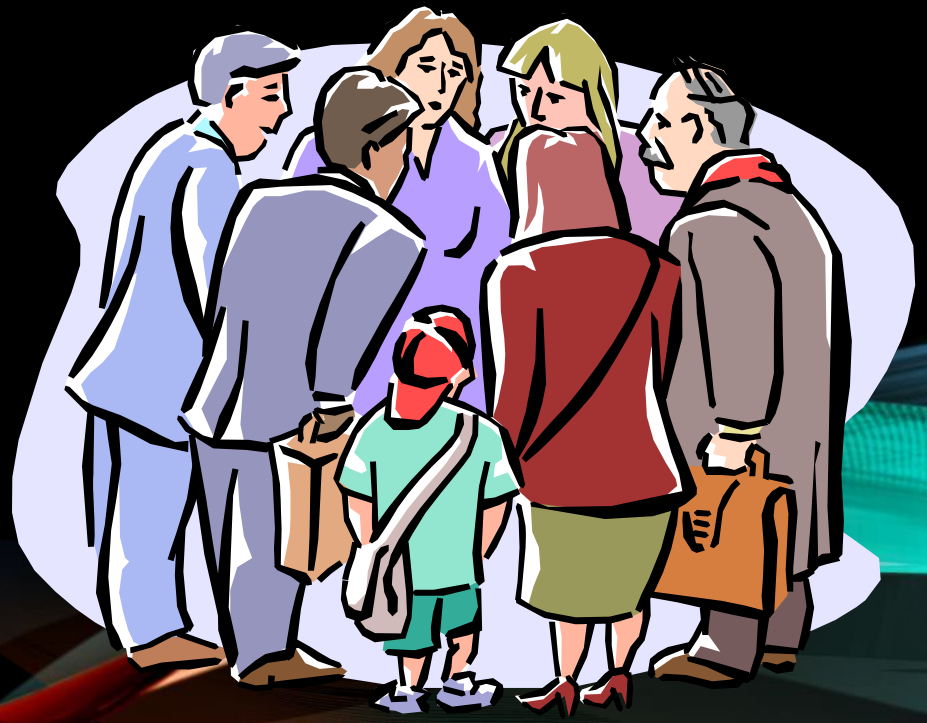
- **Factor out the GCF of**
- $\mathbf{3xy - 21y}$

- $\mathbf{5x - 35}$

DAY 2 FACTOR BY GROUPING

- **When polynomials contain four terms, it is sometimes easier to group like terms in order to factor.**
- **Your goal is to create a common factor.**
- **You can also move terms around in the polynomial to create a common factor.**
- **Practice makes you better in recognizing common factors.**

FACTORING FOUR TERM POLYNOMIALS



FACTOR BY GROUPING

EXAMPLE 1:

- **FACTOR:** $3xy - 21y + 5x - 35$
- **Factor the first two terms:**

  $3xy - 21y = 3y(x - 7)$
- **Factor the last two terms:**

  $+ 5x - 35 = 5(x - 7)$
- **The green parentheses are the same so it's the common factor**

  Now you have a common factor

$$(x - 7)(3y + 5)$$

FACTOR BY GROUPING

EXAMPLE 2:

- FACTOR: $6mx - 4m + 3rx - 2r$
- Factor the first two terms:

  $6mx - 4m = 2m(3x - 2)$
- Factor the last two terms:

  $+ 3rx - 2r = r(3x - 2)$
- The green parentheses are the same so it's the common factor

  Now you have a common factor

$$(3x - 2)\ (2m + r)$$

FACTOR BY GROUPING

EXAMPLE 3:

- **FACTOR:** $32x + 9xy + 16y + 18x^2$
- **Factor the first two terms:**

  $32x + 9xy$ **= 1 (32x + 9xy)**
- **Factor the last two terms:**

  + 16y + 18x^2 **= *1* (16y + 18**x^2**)**
- **The green parentheses are not the same, so we must reorder it**

FACTOR BY GROUPING

EXAMPLE 3:

- FACTOR: $32x + 9xy + 16y + 18x^2$
- REORDER: $32x + 16y + 9xy + 18x^2$
- Factor the first two terms:

  $32x + 16y = 16\,(2x + y)$
- Factor the last two terms:

  $+ 9xy + 18x^2 = 9x\,(y + 2x)$
- The green parentheses are the same, although a different order which is perfectly fine so we have this factorization:
- $(2x + y)(16 + 9x)$

FACTOR BY GROUPING

EXAMPLE 4:

- FACTOR: $15x - 3xy + 4y - 20$
- Factor the first two terms:

  $15x - 3xy = 3x(5 - y)$
- Factor the last two terms:

  $+ 4y - 20 = 4(y - 5)$
- The green parentheses are opposites so change the sign on the 4

  $-4(-y + 5)$ or $-4(5 - y)$
- Now you have a common factor

  $(5 - y)(3x - 4)$

WORK ON HOMEWORK

- Bring a laptop tomorrow if you have one

DAY 3 FACTORING BY GROUPING PART 2

- Grab a laptop from the laptop cart and get in groups of no more than 3.
- You can work alone if necessary
- Use your own laptop if possible
- Go to student.desmos.com
- Enter class code 3C9YD
- Drag and drop 4 term polynomials together with their factors. Once finished, work individually on homework until the end of class.

WARM UP FOR DAY 4

- Factor by grouping:
- $5x^2y - 15xy^2 - 3x + 9y$
- $10a^3b^5 + 18ab - 5a^4b^4 - 9a^2$

FACTORING CHART

THIS CHART WILL HELP YOU TO DETERMINE WHICH METHOD OF FACTORING TO USE.

TYPE	NUMBER OF TERMS
1. GCF	2 or more
2. Difference of Squares (today)	2
3. Trinomial factoring (coming up)	3
4. Grouping	4

DETERMINE THE PATTERN

1	$= 1^2$
4	$= 2^2$
9	$= 3^2$
16	$= 4^2$
25	$= 5^2$
36	$= 6^2$
…	

These are **perfect squares**!

You should be able to list the first 15 perfect squares in 30 seconds…

Perfect squares

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

REVIEW: MULTIPLY $(X - 2)(X + 2)$

	x	-2
x	x^2	-2x
+2	+2x	-4

Notice the middle terms eliminate each other!

Combine like terms.

$x^2 - 4$

This is called the <u>difference of squares</u>.

DIFFERENCE OF SQUARES

$a^2 - b^2 = (a - b)(a + b)$

or

$a^2 - b^2 = (a + b)(a - b)$

The order does <u>not</u> matter!!

4 STEPS FOR FACTORING DIFFERENCE OF SQUARES

1. Are there only 2 terms?
2. Is the first term a perfect square?
3. Is the last term a perfect square?
4. Is there subtraction (difference) in the problem?

If all of these are true, you **can** factor using this method!!!

1. FACTOR X^2 - 25

Do you have a GCF? **No**

Are the Difference of Squares steps true?

- Two terms? **Yes**
- 1st term a perfect square? **Yes**
- 2nd term a perfect square? **Yes**
- Subtraction? **Yes**
- Write your answer!

$$x^2 - 25$$

$$(x + 5)(x - 5)$$

2. FACTOR $16X^2 - 9$

<u>When factoring, use your factoring table.</u>

Do you have a GCF? **No**

Are the Difference of Squares steps true?

- Two terms? **Yes**
- 1st term a perfect square? **Yes**
- 2nd term a perfect square? **Yes**
- Subtraction? **Yes**
- Write your answer!

$$16x^2 - 9$$

$$(4x + 3)(4x - 3)$$

3. FACTOR $81A^2 - 49B^2$

<u>When factoring, use your factoring table.</u>

Do you have a GCF? **No**

Are the Difference of Squares steps true?

- Two terms? **Yes**
- 1st term a perfect square? **Yes**
- 2nd term a perfect square? **Yes**
- Subtraction? **Yes**
- Write your answer!

$$81a^2 - 49b^2$$

$(9a + 7b)(9a - 7b)$

FACTOR $X^2 - Y^2$

1. $(x + y)(x + y)$
2. $(x - y)(x + y)$
3. ✓ $(x + y)(x - y)$
4. ✓ $(x - y)(x - y)$

Remember, the order doesn't matter!

4. FACTOR $75X^2 - 12$

When factoring, use your factoring table.

Do you have a GCF?

Yes! GCF = 3

$3(25x^2 - 4)$

Are the Difference of Squares steps true?

- Two terms? **Yes**
- 1st term a perfect square? **Yes**
- 2nd term a perfect square? **Yes**
- Subtraction? **Yes**
- Write your answer!

$$3(25x^2 - 4)$$

$3(5x + 2)(5x - 2)$

FACTOR $18C^2 + 8D^2$

1. prime
2. $2(9c^2 + 4d^2)$
3. $2(3c - 2d)(3c + 2d)$
4. $2(3c + 2d)(3c + 2d)$

You cannot factor using difference of squares because there is no subtraction!

FACTOR $-64 + 4M^2$

1. prime
2. $(2m - 8)(2m + 8)$
3. $4(-16 + m^2)$
4. $4(m - 4)(m + 4)$

Rewrite the problem as $4m^2 - 64$ so the subtraction is in the middle!

WORK ON HOMEWORK FOR REST OF CLASS

DAY 5 WARMUP

- 1. $a^2 - 16$
- 2. $x^2 - 25$
- 3. $4y^2 - 16$
- 4. $9y^2 - 25$
- 5. $3r^2 - 81$
- 6. $2a^2 + 16$

FACTORING TRINOMIALS
$(ax^2 + bx + c)\ where\ a = 1$

Factoring Trinomials a=1

Again, we will *factor* trinomials such as $x^2 + 7x + 12$ back into binomials.

In this method we look for the pattern of products and sums!

If the x^2 term has no coefficient (<u>other than 1</u>)...

$$x^2 + 7x + 12$$

<u>Step 1</u>: List all pairs of numbers that multiply to equal the constant, 12.

$$12 = 1 \cdot 12$$
$$= 2 \cdot 6$$
$$= 3 \cdot 4$$

Factoring Trinomials a=1

$$x^2 + 7x + 12$$

12 = 1 • 12

1+12=13 no

= 2 • 6

2+6=8 no

= 3 • 4

3+4=7 yep

<u>Step 2</u>: Choose the pair that adds up to the middle coefficient.

<u>Step 3</u>: Fill those numbers into the blanks in the binomials:

$$(x + 3)(x + 4)$$

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

Factoring Trinomials a=1

Factor. $x^2 + 9x + 14$

<u>Step 1</u>: List all pairs of numbers that multiply to equal the constant, 14.

14 =

<u>Step 2</u>: Which pair adds up to 9?

<u>Step 3</u>: Write the binomial factors.

$$x^2 + 9x + 14 = (x + 7)(x + 2)$$

Factoring Trinomials a=1

Factor. $x^2 + 13x + 36$

<u>Step 1</u>: List all pairs of numbers that multiply to equal the constant, 36.

36 =

<u>Step 2</u>: Which pair adds up to 13?

<u>Step 3</u>: Write the binomial factors.

$$x^2 + 13x + 36 = (x + 9)(x + 4)$$

Factoring Trinomials a=1

Factor. $x^2 + 2x - 24$

This time, the constant is negative!

<u>Step 1</u>: List all pairs of numbers that multiply to equal the constant, -24. ***(To get -24, one number must be positive and one negative.)***

$$-24 = (1 \bullet -24),\ (-1 \bullet 24)$$
$$= (2 \bullet -12),\ (-2 \bullet 12)$$
$$= (3 \bullet -8),\ (-3 \bullet 8)$$
$$= (4 \bullet -6),\ (-4 \bullet 6)$$

<u>Step 2</u>: Which pair adds up to 2?

<u>Step 3</u>: Write the binomial factors.

$$x^2 + 2x - 24 = (x - 4)(x + 6)$$

Factoring Trinomials a=1

Factor. $x^2 - 3x - 18$

This time, the constant is negative!

<u>Step 1</u>: List all pairs of numbers that multiply to equal the constant, -18. ***(To get -18, one number must be positive and one negative.)***

-18 =

<u>Step 2</u>: Which pair adds up to -3?

<u>Step 3</u>: Write the binomial factors.

$$x^2 - 3x - 18 = (x - 6)(x + 3)$$

WORK ON HOMEWORK

WARMUP FOR DAY 6

- Factor the following:
- $x^2 - 7x - 18$
- $x^2 + 11x + 30$
- $x^2 - 7x + 12$
- $9x^2 - 49$

$27x^2 - 300$

$3(9x^2 - 100)$

$\boxed{3(3x+10)(3x-10)}$

$(3x+7)(3x-7)$ ☆

FACTORING TRINOMIALS WITH A > 1

Objective: To discover factoring quadratics with a leading coefficient greater than 1 and special care quadratics

$ax^2 + bx + c$

FACTORING WHEN $|A| \neq 1$

- **Step 1**: Since there is no common factors find *ac*
- **Step 2**: Rewrite *bx* as the sum of the two terms with coefficients that are factors of *ac*, and have a sum of *b*
- **Step 3**: Factor

Multiply 3 times 6.

$$3x^2 + \cancel{11}x + 6$$

To make the sum positive, both numbers must be positive.

product	sum
18	+ 11
1 • 18	+ 19
2 • 9	+ 11 ✓

$(3x^2 + 2x) + (9x + 6)$

$x(3x+2) + 3(3x+2)$

$\boxed{(3x+2)(x+3)}$

Multiply 3 times – 6.

$$3x^2 + 7x - 6$$

-18

To make the sum positive, the largest number must be positive.

product	sum
–18	+ 7
–1 • 18	+ 17
–2 • 9	+ 7 ✓

-18, 1	18, -1
-9, 2	9, -2
-6, 3	6, -3

$(3x^2 + 9x) + (-2x - 6)$

$3x(x+3) - 2(x+3)$

$(x+3)(3x-2)$

$(3x^2 - 2x) + (9x - 6)$

$x(3x-2) + 3(3x-2)$

$(3x-2)(x+3)$

Multiply 3 times – 6.

$$3x^2 - 7x - 6$$

To make the sum negative, the largest number must be negative.

product	sum
–18	– 7
1 • –18	– 17
2 • – 9	– 7 ✓

-18 , 1

-9 , 2

-6 , 3

18 , -1

9 , -2

6 , -3

$(3x^2 - 9x) + (2x - 6)$

$3x(\cancel{x-3}) + 2(\cancel{x-3})$

$(x-3)(3x+2)$

Check using the F.O.I.L. method:

$(3x + 2)(x - 3)$

$3x^2 - 9x + 2x - 6$

$3x^2 - 7x - 6$

Multiply 9 times 4.

$$9x^2 - 12x + 4$$

To make the sum negative, both numbers must be negative.

product	sum
36	– 12
–1 • –36	– 37
–2 • –18	– 20
–3 • –12	– 15
–4 • –9	– 13
–6 • –6	– 12 ✓

$(9x^2 - 6x)(-6x + 4)$

$3x(3x-2) - 2(3x-2)$

$(3x-2)(3x-2)$

Check using the F.O.I.L. method:

$(3x - 2)(3x - 2)$

$9x^2 - 6x - 6x + 4$

$9x^2 - 12x + 4$ ✓

PERFECT SQUARE TRINOMIALS

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

$4x^2 - 24x + 36$

$x^2 + 10x + 25$

$9x^2 - 48x + 64$

$49x^2 + 56x + 16$

FACTORING A DIFFERENCE OF SQUARES

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^2 - 16$$

$$4x^2 - 9$$

$$25x^2 - 49$$

$$81x^2 - 100$$

ALWAYS

check your work.

FACTORING WITH A LEADING COEF

$3X^2 - 11X - 20$

-60

60, −1	−60, 1
30, −2	−30, 2
20, −3	−20, 3
15, −4	−15, 4
12, −5	−12, 5
10, −6	−10, 6

$(3x^2 - 15x) + (4x - 20)$

$3x(x-5) + 4(x-5)$

$(x-5)(3x+4)$

-120

$-120, 1 \ldots$

$-12, 10$ or $10, -12$

$8X^2 - 2X - 15$

$(8x^2 - 12x) + (10x - 15)$

$4x(2x-3) + 5(2x-3)$

$\boxed{(2x-3)(4x+5)}$

$4x^2 - 64$

$9x^2 - 100$

$3x^2 + 21x + 36$

$6x^2 + 27x - 15$