Math III UNIT 3 OVERVIEW: Modeling with Polynomial Functions

Unit Outcomes At the end of this unit, your student should be able to:	**Key Vocabulary** Terms to deepen the student's understanding
✓ Explain why the sum or product of two rational numbers is rational. ✓ Explain why the sum of a rational number and an irrational number is irrational. ✓ Explain why the product of a nonzero rational number and an irrational number is irrational. ✓ Add, subtract, and multiply complex numbers. ✓ Solve quadratic equations with real coefficients that have complex solutions. ✓ Solve quadratic equations by inspection, taking square roots, factoring, completing the square, and using the quadratic formula. ✓ Determine which method for solving a quadratic equation is most appropriate based on the initial form of the equation. ✓ Derive the quadratic formula using the process of completing the square. ✓ Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. ✓ Show that the Fundamental Theorem of Algebra is true for quadratic polynomials. ✓ Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. ✓ Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. ✓ Derive the equation of a parabola given a focus and directrix. ✓ Add, subtract, and multiply polynomials. ✓ Solve polynomial equations and systems of polynomial equations approximately by using technology to graph the functions they define. ✓ Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph showing key features of the function defined by the polynomial. Key features include intercepts, relative maxima and minima, and end behavior. ✓ Prove polynomial identities and use them to describe numerical relationships.	✓ Rational Number ✓ Irrational Number ✓ Imaginary Number ✓ Complex Number ✓ Polynomial ✓ Degree ✓ Quadratic Formula ✓ Discriminant ✓ Completing the Square ✓ Vertex Form ✓ Standard Polynomial Form ✓ Local Maximum ✓ Local Minimum ✓ Maximum ✓ Minimum ✓ X-intercept ✓ Zero ✓ Vertex ✓ End Behavior ✓ Cubic ✓ Quadratic ✓ Factor ✓ Linear Term ✓ Focus ✓ Directrix ✓ Parabola ✓ Fundamental Theorem of Algebra ✓ Polynomial Identity

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Key Standards Addressed Connections to Common Core/NC Essential Standards	**Where This Unit Fits** Connections to prior and future learning
N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **N-CN.1** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **N-CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N-CN.7** Solve quadratic equations with real coefficients that have complex solutions. **N-CN.9** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **A-APR.3** Identify zeros of polynomials when suitable factorizations are	**Coming into this unit, students should have a strong foundation in:** ✓ The real number system ✓ Factoring quadratic expressions ✓ Adding and subtracting quadratic expressions; multiplying binomials ✓ Solving quadratic equations by factoring and using the quadratic formula ✓ Analyzing functions using different representations ✓ The relationship between zeros and factors ✓ Representing and solving equations graphically using technology **This unit builds to the following future skills and concepts:** **Pre-Calculus** **2.01** Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined & greatest integer to model and solve problems; justify results. **2.03** For sets of data, create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, logistic, and logarithmic functions. **2.08** Explore the limit of a function graphically, numerically, and algebraically.

available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

A-REI.4 Solve quadratic equations in one variable.

a) Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b) Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, **polynomial**, rational, absolute value, exponential, and logarithmic functions.

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

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F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a) Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **G-GPE.2** Derive the equation of a parabola given a focus and directrix.	

Additional Resources

Materials to support understanding and enrichment

- ✓ Operations with Complex Numbers
- ✓ Complex Numbers
- ✓ Solving Quadratic Equations by Factoring
- ✓ Solving Quadratic Equations with Complex Roots
- ✓ What is the Discriminant?
- ✓ Using the Discriminant to Determine the Type of Solutions
- ✓ Solving Quadratics using the Quadratic Formula
- ✓ Solving Quadratic Equations with Complex Solutions using the Quadratic Formula
- ✓ Completing the Square
- ✓ Completing the Square (video)
- ✓ Completing the Square for Vertex Form
- ✓ Deriving the Quadratic Formula by Completing the Square
- ✓ Using the Focus and Directrix to find the Equation of a Parabola
- ✓ Finding the Vertex of a Quadratic Function
- ✓ Finding the Vertex of a Quadratic Function by Completing the Square
- ✓ Standard Form of a Polynomial
- ✓ Finding the Zeros of a Polynomial Function from a Table of Values
- ✓ End Behavior of Polynomials
- ✓ Graphing Polynomials in Factored Form
- ✓ Fundamental Theorem of Algebra
- ✓ Fundamental Theorem of Algebra (video)

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- ✓ Axis of Symmetry
- ✓ Proofs of Polynomial Identities

*** Please note**, the unit guides are a work in progress. If you have feedback or suggestions on improvement, please feel free to contact sdupree@wcpss.net.