Transformations of Quadratic Functions (from Vertex Form)

Parent Function: $y = x^2$

Minimum: $a>0$
maximum: $a<0$

Vertex: (h,k)

horizontal translations
$x-h$ right
$x+h$ left

$$y = a(x-h)^2 + k$$

vertical translations
$+k$ up
$-k$ down

$0<|a|<1$ vertical shrink
$|a|>1$ vertical stretch
$-a$ reflect over x-axis

Examples: List the vertex, state minimum or maximum, transformations, domain, range, x-intercept, y-intercept, increasing interval, decreasing interval, and end behavior.

1. $y = 2x^2$

2. $y = \frac{1}{3}x^2$

3. $y = -x^2$

4. $y = (x-3)^2 + 2$

5. $y = (x+4)^2 - 1$

6. $y = 4(x-2)^2 - 3$

7. $y = -3(x+1)^2 - 5$

8. $y = -\frac{2}{3}(x+3)^2 + 12$

9. $y = -4x^2 + 10$

10. $y = \frac{1}{2}(x-5)^2$

Quadratic Functions – Parent Function $y = x^2$

Function	Vertex	Min or Max?	Transformations	Domain	Range	x-intercept	y-intercept	Increasing Interval	Decreasing Interval	End Behavior
1.	(0,0)	min	vertical stretch of 2	$(-\infty,\infty)$	$[0,\infty)$	(0,0)	(0,0)	$(0,\infty)$	$(-\infty,0)$	As $x\to\infty, y\to\infty$. As $x\to-\infty, y\to\infty$.
2.	(0,0)	min	vertical shrink of $\frac{1}{3}$	$(-\infty,\infty)$	$[0,\infty)$	(0,0)	(0,0)	$(0,\infty)$	$(-\infty,0)$	As $x\to\infty, y\to\infty$. As $x\to-\infty, y\to\infty$.
3.	(0,0)	max	reflect over x-axis	$(-\infty,\infty)$	$(-\infty,0]$	(0,0)	(0,0)	$(-\infty,0)$	$(0,\infty)$	As $x\to\infty, y\to-\infty$. As $x\to\infty, y\to-\infty$
4.	(3,2)	min	right 3, up 2	$(-\infty,\infty)$	$[2,\infty)$	none	(0,11)	$(3,\infty)$	$(-\infty,3)$	As $x\to-\infty, y\to\infty$. As $x\to\infty, y\to\infty$.
5.	(-4,-1)	min	left 4, down 1	$(-\infty,\infty)$	$[-1,\infty)$	(-5,0) (-3,0)	(0,15)	$(-4,\infty)$	$(-\infty,-4)$	As $x\to-\infty, y\to\infty$. As $x\to\infty, y\to\infty$.
6.	(2,-3)	min	vertical stretch of 4, right 2, down 3	$(-\infty,\infty)$	$[-3,\infty)$	≈ (1.13,0) (2.87,0)	(0,13)	$(2,\infty)$	$(-\infty,2)$	As $x\to-\infty, y\to\infty$. As $x\to\infty, y\to\infty$.
7.	(-1,-5)	max	reflect over x-axis, vertical stretch of 3, left 1, down 5	$(-\infty,\infty)$	$(-\infty,-5]$	none	(0,-8)	$(-\infty,-1)$	$(-1,\infty)$	As $x\to-\infty, y\to-\infty$. As $x\to\infty, y\to-\infty$.
8.	(-3,12)	max	reflect over x-axis, vertical shrink of $\frac{2}{3}$, left 3, up 12.	$(-\infty,\infty)$	$(-\infty,12]$	≈ (-7.24,0) (1.24,0)	(0,6)	$(-\infty,-3)$	$(-3,\infty)$	As $x\to-\infty, y\to-\infty$. As $x\to\infty, y\to-\infty$.
9.	(0,10)	max	reflect over x-axis, vertical stretch of 4, up 10	$(-\infty,\infty)$	$(-\infty,10]$	≈ (-1.58,0) (1.58,0)	(0,10)	$(-\infty,0)$	$(0,\infty)$	As $x\to-\infty, y\to-\infty$. As $x\to\infty, y\to-\infty$.
10.	(5,0)	min	vertical shrink of $\frac{1}{2}$, right 5	$(-\infty,\infty)$	$[0,\infty)$	(5,0)	(0,12.5)	$(5,\infty)$	$(-\infty,5)$	As $x\to-\infty, y\to\infty$. As $x\to\infty, y\to\infty$.

Transformations of Square Root Functions

Parent Function: $y = \sqrt{x}$

starting point: (h, k)

horizontal translations
x-h right
x+h left

$$y = a\sqrt{x - h} + k$$

vertical translations
+k up
-k down

$0<|a|<1$ vertical shrink
$|a|>1$ vertical stretch
$-a$ reflect over x-axis

Examples: List the starting point, transformations, domain, range, x-intercept, y-intercept, increasing interval, decreasing interval, and end behavior.

1. $y = 2\sqrt{x}$

2. $y = \frac{1}{3}\sqrt{x}$

3. $y = -\sqrt{x}$

4. $y = \sqrt{x-3} + 2$

5. $y = \sqrt{x+4} - 1$

6. $y = 4\sqrt{x-2} - 3$

7. $y = -3\sqrt{x+1} - 5$

8. $y = -\frac{2}{3}\sqrt{x+3} + 12$

9. $y = -4\sqrt{x} + 10$

10. $y = \frac{1}{2}\sqrt{x-5}$

Notice that these functions do not have a vertex!

Square Root Functions – Parent Function $y = \sqrt{x}$

Function	First Point	Min or Max?	Transformations	Domain	Range	x-intercept	y-intercept	Increasing Interval	Decreasing Interval	End Behavior	
1.	(0,0)	min	vertical stretch of 2	$[0,\infty)$	$[0,\infty)$	(0,0)	(0,0)	$(0,\infty)$	never	As $x \to 0$, $y \to 0$. As $x \to \infty$, $y \to \infty$.	
2.	(0,0)	min	vertical shrink of 1/3	$[0,\infty)$	$[0,\infty)$	(0,0)	(0,0)	$(0,\infty)$	never	As $x \to \infty$, $y \to \infty$.	As $x \to 0$, $y \to 0$
3.	(0,0)	max	reflection over x-axis	$[0,\infty)$	$(-\infty,0]$	(0,0)	(0,0)	never	$(0,\infty)$	As $x \to \infty$, $y \to -\infty$.	As $x \to 0$, $y \to 0$
4.	(3,2)	min	right 3, up 2	$[3,\infty)$	$[2,\infty)$	none	none	$(3,\infty)$	never	As $x \to \infty$, $y \to \infty$.	As $x \to 3$, $y \to 2$
5.	(-4,-1)	min	left 4, down 1	$[-4,\infty)$	$[-1,\infty)$	(-3,0)	(0,1)	$(-4,\infty)$	never	As $x \to \infty$, $y \to \infty$.	As $x \to -4$, $y \to -1$
6.	(2,-3)	min.	vertical stretch of 4, right 2, down 3	$[2,\infty)$	$[-3,\infty)$	≈(2.56, 0)	none	$(2,\infty)$	never	As $x \to \infty$, $y \to \infty$.	As $x \to 2$, $y \to -3$
7.	(-1,-5)	max	reflect over x-axis, vertical stretch of 3, left 1, down 5	$[-1,\infty)$	$(-\infty,-5]$	none	(0,-8)	never	$(-1,\infty)$	As $x \to \infty$, $y \to -\infty$.	As $x \to -1$, $y \to -5$
8.	(-3,12)	max	reflect over x-axis, vertical shrink of 4/3, left 3, up 12	$[-3,\infty)$	$(-\infty,12]$	(324,0)	(0,10.85)	never	$(-3,\infty)$	As $x \to \infty$, $y \to -\infty$.	As $x \to -3$, $y \to 12$
9.	(0,10)	max	reflect over x-axis, vertical stretch of 4, up 10	$[0,\infty)$	$(-\infty,0]$	(6.25, 0)	(0,10)	never	$(0,\infty)$	As $x \to \infty$, $y \to -\infty$.	As $x \to 0$, $y \to 10$
10.	(5,0)	min	vertical shrink of $\frac{1}{2}$, right 5	$[5,\infty)$	$[0,\infty)$	(5,0)	none	$(5,\infty)$	never	As $x \to \infty$, $y \to \infty$.	As $x \to 5$, $y \to 0$

Transformations of Inverse Variation Functions

Parent Function: $y = \frac{1}{x}$

intersection of asymptotes: (h, k)

Vertical asymptote: x = h

horizontal asymptote: y = k

$$y = \frac{a}{x - h} + k$$

$0 < |a| < 1$ vertical shrink (remember fractions use numerator + denominator $y = \frac{2}{3x}$)

$|a| > 1$ vertical stretch

$-a$ reflect over x-axis

$+k$ vertical translations

+K up

−K down

horizontal translations

x − h right

x + h left

Examples: List the transformations, domain, range, horizontal asymptote, vertical asymptote, increasing interval, decreasing interval, and end behavior.

1. $y = \frac{2}{x}$

2. $y = \frac{1}{3x}$

3. $y = -\frac{1}{x}$

4. $y = \frac{1}{x-3} + 2$

5. $y = \frac{1}{x+4} - 1$

6. $y = \frac{4}{x-2} - 3$

7. $y = -\frac{3}{x+1} - 5$

8. $y = -\frac{2}{3(x+3)} + 12$

9. $y = -\frac{4}{x} + 10$

10. $y = \frac{1}{2(x-5)}$

Inverse Variation Functions – Parent Function $y = \frac{1}{x}$

Function	Min or Max?	Transformations	Domain	Range	Horizontal Asymptote	Vertical Asymptote	Increasing Interval	Decreasing Interval	End Behavior
1.	none	vertical stretch of 2	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0)\cup(0,\infty)$	$y=0$	$x=0$	never	$(-\infty,0)\cup(0,\infty)$	As $x\to-\infty, y\to0$. As $x\to\infty, y\to0$.
2.	none	vertical shrink of $\frac{1}{3}$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0)\cup(0,\infty)$	$y=0$	$x=0$	never	$(-\infty,0)\cup(0,\infty)$	As $x\to-\infty, y\to0$. As $x\to\infty, y\to0$.
3.	none	reflect over x-axis	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0)\cup(0,\infty)$	$y=0$	$x=0$	$(-\infty,0)\cup(0,\infty)$	never	As $x\to\infty, y\to0$. As $x\to-\infty, y\to0$.
4.	none	right 3, up 2	$(-\infty,3)\cup(3,\infty)$	$(-\infty,2)\cup(2,\infty)$	$y=2$	$x=3$	never	$(-\infty,3)\cup(3,\infty)$	As $x\to-\infty, y\to2$. As $x\to\infty, y\to2$.
5.	none	left 4, down 1	$(-\infty,-4)\cup(-4,\infty)$	$(-\infty,-1)\cup(-1,\infty)$	$y=-1$	$x=-4$	never	$(-\infty,-4)\cup(-4,\infty)$	As $x\to\infty, y\to-1$. As $x\to-\infty, y\to-1$.
6.	none	vertical stretch of 4, right 2, down 3	$(-\infty,2)\cup(2,\infty)$	$(-\infty,-3)\cup(-3,\infty)$	$y=-3$	$x=2$	never	$(-\infty,2)\cup(2,\infty)$	As $x\to-\infty, y\to-3$. As $x\to\infty, y\to-3$.
7.	none	reflect over x-axis, vertical stretch of 3, left 1, down 5	$(-\infty,-1)\cup(-1,\infty)$	$(-\infty,-5)\cup(-5,\infty)$	$y=-5$	$x=-1$	$(-\infty,-1)\cup(-1,\infty)$	never	As $x\to-\infty, y\to-5$. As $x\to\infty, y\to-5$.
8.	none	reflect over x-axis, vertical shrink of $2/3$, left 3, up 12	$(-\infty,-3)\cup(-3,\infty)$	$(-\infty,12)\cup(12,\infty)$	$y=12$	$x=-3$	$(-\infty,-3)\cup(-3,\infty)$	never	As $x\to-\infty, y\to12$. As $x\to\infty, y\to12$.
9.	none	reflect over x-axis, vertical stretch of 4, up 10	$(-\infty,0)\cup(0,\infty)$	$(-\infty,10)\cup(10,\infty)$	$y=10$	$x=0$	$(-\infty,0)\cup(0,\infty)$	never	As $x\to-\infty, y\to10$. As $x\to\infty, y\to10$.
10.	none	vertical shrink of $\frac{1}{2}$, right 5	$(-\infty,5)\cup(5,\infty)$	$(-\infty,0)\cup(0,\infty)$	$y=0$	$x=5$	never	$(-\infty,5)\cup(5,\infty)$	As $x\to-\infty, y\to0$. As $x\to\infty, y\to0$.