Math III Unit 3 Part 1: QUADRATIC MODELING AND EQUATIONS

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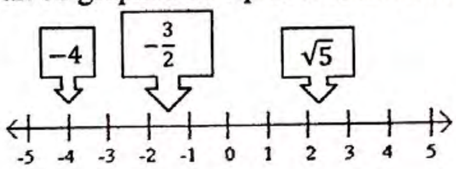
Main topics of instruction:

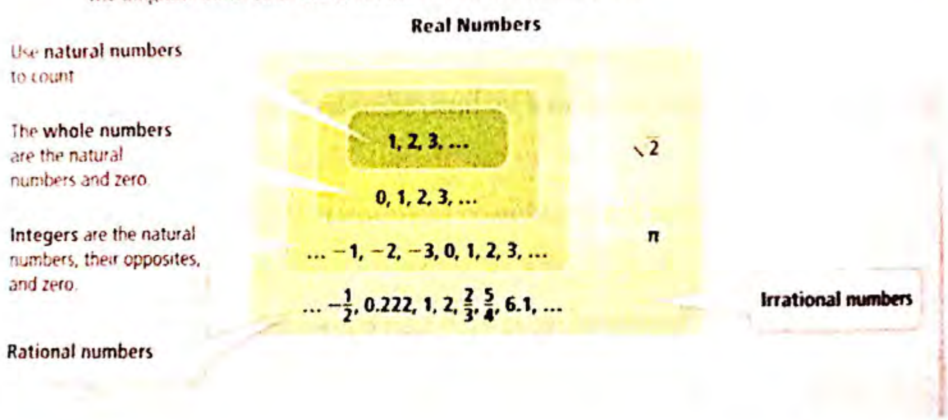
1) The Real Number System
2) Factoring and solving quadratic equations
3) Graphing quadratic equations
4) Complex Numbers

Day 1: The Real Number System and Factoring

There are two types of real numbers: rational numbers and irrational numbers. Every real number can be graphed as a point on the number line.



The diagram shows how subsets of the real numbers are related.



Rational Numbers

- Can be expressed as a quotient of integers (a fraction)
- Can be a terminating or repeating decimal
- Includes all integers

Irrational Numbers

- Cannot be expressed as a fraction
- Decimals do not repeat or terminate

Classify the following as rational or irrational. If a number is rational, state if it is a natural number, whole number, integer, or simply rational.

a) 4 rational, natural, whole, integer

b) -3 rational, integer

c) $\sqrt{6}$ irrational

d) 0.125 rational

e) $\frac{-2}{5}$ rational

f) $\sqrt{\frac{1}{4}}$ rational

g) 0 rational, whole, integer

f) π irrational

Critical Thinking: In each scenario, answer Always, Sometimes, or Never. If the answer is Sometimes, give examples of each outcome.

a) The sum of a rational number and a rational number is a rational number.

Always

b) The product of two rational numbers is a rational number.

Always

c) The sum of a rational number and an irrational number is an irrational number.

Always

d) The product of a rational number and an irrational number is an irrational number.

Sometimes $(3 \cdot \sqrt{2} = 3\sqrt{2},\ 0 \cdot \pi = 0)$

e) The sum of two irrational numbers is an irrational number.

$6\sqrt{7} + (-6\sqrt{7}) = 0$ Sometimes

f) The product of two irrational numbers is an irrational number.

Sometimes $(\sqrt{2} \cdot \sqrt{3} = \sqrt{6},\ \sqrt{\frac{9}{2}} \cdot \sqrt{2} = \sqrt{9} = \pm 3)$

Factoring – Quadratics

Greatest Common Factor (GCF): The largest term that can be pulled out of an expression.

Example 1:

Factor and solve $4w^2 + 2w = 0$

$2w(2w+1) = 0$

$2w = 0$ | $2w + 1 = 0$

$\boxed{w=0}$ | $2w = -1$

$\boxed{w = -\frac{1}{2}}$

You try!

Factor and solve $5x^2 - 15x = 0$

$5x(x-3) = 0$

$5x = 0$ | $x - 3 = 0$

$\boxed{x=0}$ | $\boxed{x=3}$

Standard form of a quadratic expression: $y = ax^2 + bx + c$

Example 2: Factor and solve $x^2 + 8x + 7 = 0$

7

$7, 1$

$(x^2 + 7x)(+x + 7) = 0$

$x(x+7) + 1(x+7) = 0$

$(x+1) \mid (x+7) = 0$

$x+1=0 \qquad x+7=0$

$\boxed{x=-1} \qquad \boxed{x=-7}$

You try! Factor and solve the following.

a) $x^2 - 17x + 72 = 0$

72

$-9, -8$

$(x^2 - 9x)(-8x + 72) = 0$

$x(x-9) - 8(x-9) = 0$

$(x-8) \mid (x-9) = 0$

$x-8=0 \qquad x-9=0$

$\boxed{x=8} \qquad \boxed{x=9}$

b) $3x^2 - 16x + 5 = 0$

15

$-15, -1$

$(3x^2 - 15x)(-x + 5) = 0$

$3x(x-5) - 1(x-5) = 0$

$(3x-1)(x-5) = 0$

$3x-1=0 \qquad x-5=0$

$3x=1$

$\boxed{x=\frac{1}{3}} \qquad \boxed{x=5}$

c) $9x^2 - 16$

$(3x+4)(3x-4) = 0$

$3x+4=0 \qquad 3x-4=0$

$3x=-4 \qquad 3x=4$

$\boxed{x=-\frac{4}{3}} \qquad \boxed{x=\frac{4}{3}}$

Quick! Throw these in a calculator! What do you notice about where the parabolas cross the x-axis?

They cross at the solutions we found.

These are called **zeros!** They are also called roots, solutions, x-intercepts

Factoring – Polynomials

Example 1: Factoring Using the GCF

Factor and solve $2x^3 - 22x^2 + 48x = 0$

$2x(x^2 - 11x + 24) = 0$

24

$-8, -3$

$2x(x^2 - 8x - 3x + 24) = 0$

$2x \quad x(x-8) - 3(x-8) = 0$

$2x \mid (x-3) \mid (x-8) = 0$

$2x=0 \qquad x-3=0 \qquad x-8=0$

$\boxed{x=0} \qquad \boxed{x=3} \qquad \boxed{x=8}$

You try!

Factor and solve $3x^3 + 15x^2 - 42x = 0$

$3x(x^2 + 5x - 14) = 0$

-14

$7, -2$

$3x(x^2 + 7x - 2x - 14) = 0$

$3x \quad x(x+7) - 2(x+7) = 0$

$3x \mid (x-2) \mid (x+7) = 0$

$3x=0 \qquad x-2=0 \qquad x+7=0$

$\boxed{x=0} \qquad \boxed{x=2} \qquad \boxed{x=-7}$

Example 2: Factoring Using Grouping

Factor and solve $(2x^3 - 3x^2)(-8x + 12) = 0$

$x^2(2x-3) - 4(2x-3) = 0$

$(x^2-4)(2x-3) = 0$

$(x+2)(x-2)(2x-3) = 0$

$x+2=0$ $\quad$ $x-2=0$ $\quad$ $2x-3=0$

$\boxed{x=-2}$ $\quad$ $\boxed{x=2}$ $\quad$ $2x=3$

$\boxed{x=3/2}$

You try!

Factor and solve $(3x^2 + 12x^2)(-2x - 8) = 0$

$3x(x+4) - 2(x+4) = 0$

$(3x-2)(x+4) = 0$

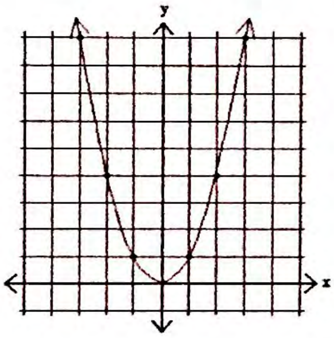
$3x-2=0$ $\quad$ $x+4=0$

$3x=2$ $\quad$ $\boxed{x=-4}$

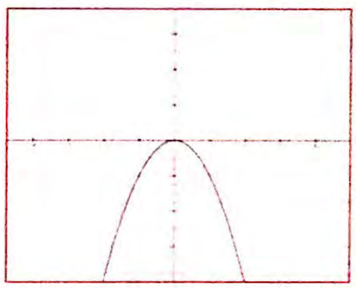
$\boxed{x=\frac{2}{3}}$

Day 2: Finding the Equation of a Parabola in Standard Form

The graph of a quadratic function is called a parabola.



$y = x^2$



$y = -x^2$

Standard Form of a Quadratic Function: $y=ax^2+bx+c$

Axis of Symmetry: $x = \frac{-b}{2a}$ line that runs down the middle (through the vertex) of the parabola.

Can be found with the formula:

Vertex: Highest or lowest (maximum or minimum) point on the parabola

How can I find the y value of the vertex? plug x back in once you find it using $x = \frac{-b}{2a}$

Example 1: Find the vertex and axis of symmetry, then graph $y = x^2 + 2x + 3$

a 1

b 2

c 3

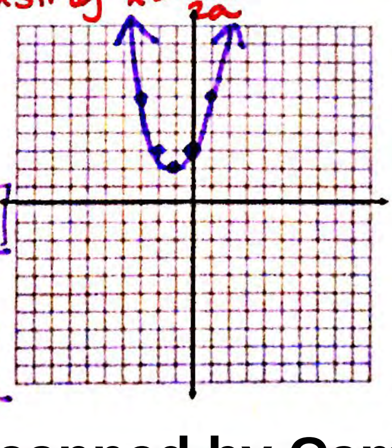
$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$

$\boxed{\text{AOS } x=-1}$

$y=(-1)^2+2(-1)+3$

$y=2$

$\boxed{\text{vertex: } (-1,2)}$



x	y
0	3
1	6
-1	2

You try! Find the vertex and axis of symmetry, then graph $y = -2x^2 + 6x - 4$.

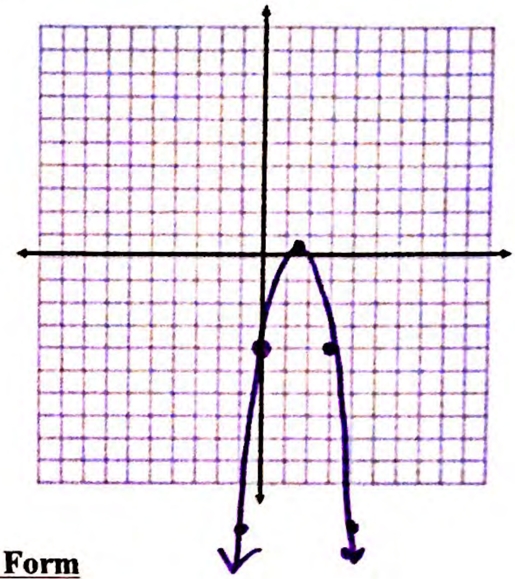
$x = \frac{-6}{2(-2)} = \frac{-6}{-4} = \frac{3}{2}$

$y = -2\left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right) - 4$

$= -4.5 + 9 - 4$

$= 0.5 = \frac{1}{2}$

vertex : $\left(\frac{3}{2}, \frac{1}{2}\right)$



x	y
0	-4
3	-4
4	-12

Finding a Quadratic Equation in Standard Form

Example 2: A parabola has three points: (2, 3), (3, 13), and (4, 29). Find a quadratic equation (model) in standard form that will fit the parabola.

Plug points into QuadReg, round to 2 decimal places

$\boxed{y = 3x^2 - 5x + 1}$

You try! A parabola has three points: (1, 0), (2, -3), and (3, -10). Find a quadratic equation (model) in standard form that will fit the parabola.

$\boxed{y = -2x^2 + 3x - 1}$

Example 3: Anthony throws a football across the field while standing on top of the bleachers. The data that follows gives the height of the ball in feet versus the seconds since the ball was thrown.

time	.2	.6	1	1.2	1.5	2	2.5	2.8	3.4	3.8	4.5
height	92	110	130	134	142	144	140	132	112	90	44

Write a quadratic model for this data. (Round to two decimal places.)

Plug points into QuadReg, round to 2 decimal places.

$\boxed{y = -16.14x^2 + 64.37x + 79.78}$

Day 3: Vertex Form and Translating Parabolas

Standard Form of a Parabola: $y = ax^2 + bx + c$

Vertex Form of a Parabola: $y = a(x-h)^2 + k$ where the vertex is (h, k).

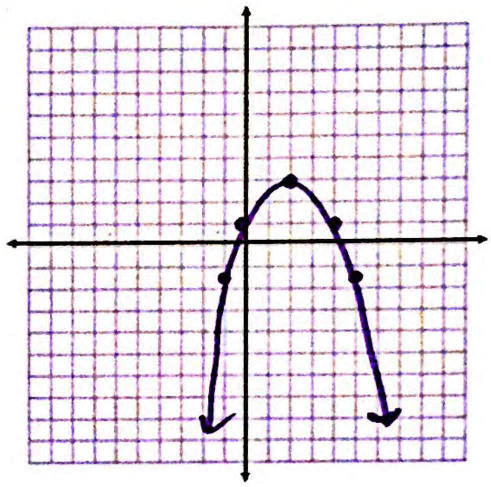
h is the opposite of what it looks like in the equation!

Using Vertex Form to Graph

Example 1: Graph $y = -\frac{1}{2}(x-2)^2 + 3$.

Where is the vertex? $(2,3)$

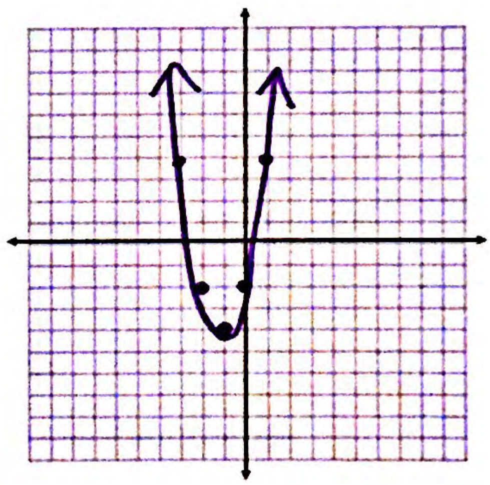
x	y
2	3
0	1
-1	-1.5



You try! Graph $y = 2(x+1)^2 - 4$.

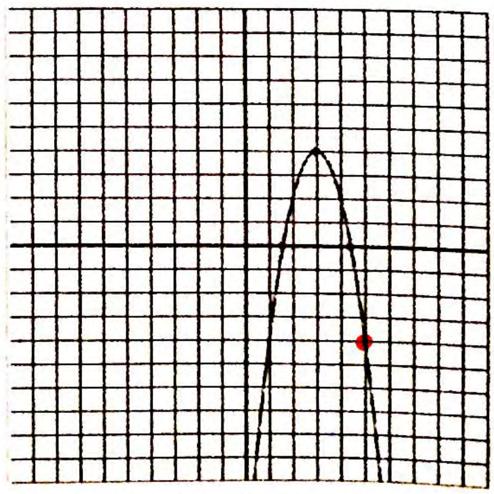
Where is the vertex? $(-1,-4)$

x	y
-1	-4
0	-2
1	4



Writing the Equation of a Parabola in Vertex Form

Example 2: Write the equation of the parabola given the graph.



Step 1: Plug the vertex into vertex form.

$y = a(x-3)^2 + 4$

Step 2: Use one other point to solve for a. $(5,-4)$

$-4 = a(5-3)^2 + 4$

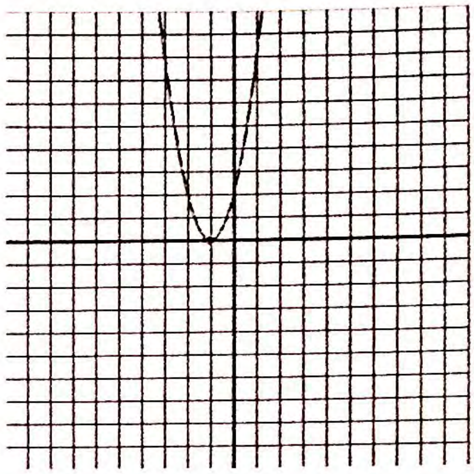
$-4 = 4a + 4$

$-8 = 4a$

$-2 = a$

$\boxed{y = -2(x-3)^2 + 4}$

You try! Write the equation of the parabola given the graph.



$y = a(x+1)^2 + 0$

plug in $(0,2)$

$2 = a(0+1)^2 + 0$

$2 = a$

$\boxed{y = 2(x+1)^2}$

Converting from Standard Form to Vertex Form

Example 3: Convert $y = 4x^2 + 26$ to vertex form.

Step 1: Find the vertex. a $\underline{4}$ b $\underline{0}$ c $\underline{26}$ $x = \frac{-0}{8} = 0$ $y = 4(0)^2 + 26$ $y = 26$ $\boxed{(0, 26)}$

Step 2: Plug the vertex into vertex form and pull *a* from the standard form equation.

$y = 4(x-0)^2 + 26 \rightarrow \boxed{y = 4(x)^2 + 26}$

You try! Convert $y = 2x^2 - 6x + 3$ to vertex form.

a $\underline{2}$ b $\underline{-6}$ c $\underline{3}$ $x = \frac{6}{4} = \frac{3}{2}$ vertex: $\left(\frac{3}{2}, -\frac{3}{2}\right)$

$y = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 3 = -\frac{3}{2}$ $\boxed{y = 2\left(x - \frac{3}{2}\right)^2 - \frac{3}{2}}$

Critical Thinking: How would you convert from vertex form back to standard form?

Multiply it out! make up a vertex form... $y = 3(x-2)^2 + 4$

$y = 3(x-2)(x-2) + 4$

$y = 3(x^2 - 4x + 4) + 4$

$y = 3x^2 - 12x + 12 + 4$

$\boxed{y = 3x^2 - 12x + 16}$

Identifying Translations of Parabolas from Vertex Form

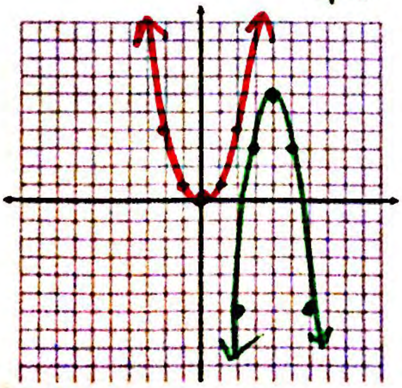
Graph $y = x^2$, then graph $y = -3(x-4)^2 + 6$.

What is different about the two graphs?

flipped, thinner, right 4, up 6, vertex: $(4, 6)$

$y = x^2$

x	y
0	0
1	1
2	4



x	y
4	6
3	3
2	-6

Rules for Transformations:

*I*nside the parentheses → *I*nverse of what you think
- Negative # → move right
- Positive # → move left

*O*utside the parentheses → *O*bvious movement
- Negative # → move down
- Positive # → move up

Negative coefficient → flipped upside down

Integer coefficient → thinner

Coefficient $0 < a < 1$ (fraction) → wider

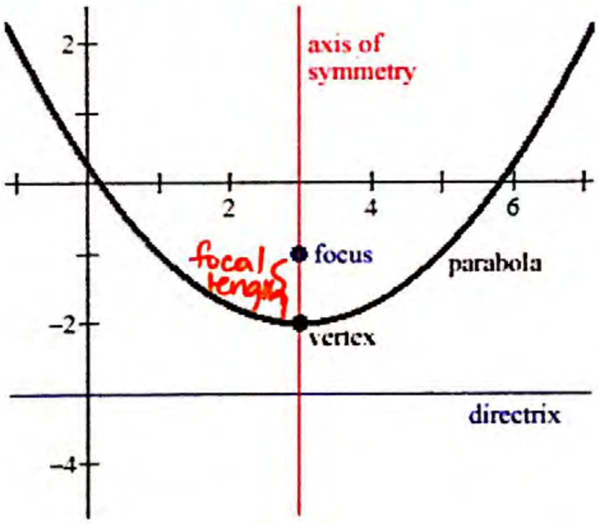
Day 4: Focus and Directrix

A parabola has two more important features known as the focus and the directrix.

Focus: A fixed point on the interior of a parabola. It is equidistant from the parabola as the directrix.

Directrix: A fixed line below or above the parabola. The distance to the parabola is the same as the distance from the focus to the parabola.

The distance between the vertex and the focus is called the focal length (c)



Example 1: Find the equation of the parabola with vertex at the origin and focus (0, 2).

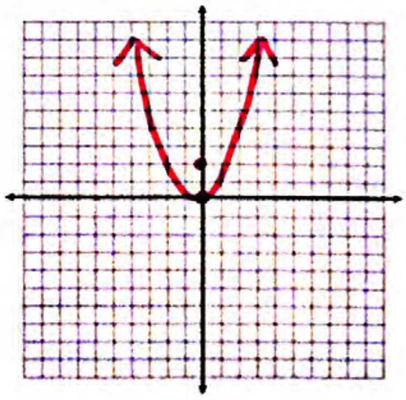
The focus is $(0, c)$, so $c=2$

$$y = \frac{1}{4c}x^2$$

$$y = \frac{1}{4(2)}x^2$$

$$\boxed{y = \frac{1}{8}x^2}$$

Draw a picture first!



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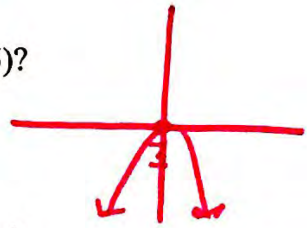
Example 2: What are the focus and directrix of the parabola with equation $y = -\frac{1}{12}x^2$?

$$a = \frac{1}{4c} = -\frac{1}{12}$$

$$4c = -12$$

$$c = -3 \rightarrow \boxed{\text{focus is } (0,-3)}$$

You try! a) What is the equation of a parabola with vertex at (0, 0) and focus (0, -1.5)?



$$y = \frac{1}{4(-1.5)}x^2$$

$$\boxed{y = \frac{1}{-6}x^2}$$

b) What are the vertex, focus, and directrix of the parabola with equation $y = \frac{x^2}{4}$?

$$y = \frac{1}{4}x^2$$

$$4 = 4c$$

$$c = 1$$

focus: $(0,1)$

vertex: $(0,0)$

directrix: $y = -1$

Example 3: What are the vertex, focus, and directrix of the parabola with equation $y = x^2 - 4x + 8$?

First, get the equation in vertex form!

$$x = \frac{-b}{2a} = \frac{4}{2(1)} = 2$$

$$y = (2)^2 - 4(2) + 8$$

$$y = 4$$

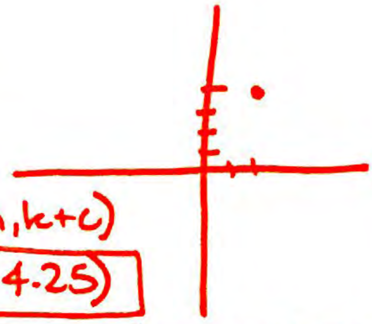
$$\boxed{\text{vertex: } (2,4)}$$

$$y = (x-2)^2 + 4$$

$$1 = \frac{1}{4c}$$

$$4c = 1$$

$$c = \frac{1}{4}$$



focus $(h, k+c)$

$= \boxed{(2, 4.25)}$

directrix: $y = k - c = 3.75$

$\boxed{y = 3.75}$

You try! a) What are the vertex, focus, and directrix of the parabola with equation $y = x^2 + 8x + 18$?

$$x = \frac{-8}{2(1)} = -4$$

$$y = (-4)^2 + 8(-4) + 18$$

$$y = 2 \quad \boxed{\text{vertex: } (-4,2)}$$

$$y = (x+4)^2 + 2$$

$$1 = \frac{1}{4c}$$

$$4c = 1$$

$$c = \frac{1}{4} \rightarrow \boxed{\text{focus: } (-4, 2.25)}$$

directrix: $2 - .25$

$= 1.75$

$\boxed{y = 1.75}$

b) What are the vertex, focus, and directrix of the parabola with equation $y = 2x^2 + 4x - 2$?

$$x = \frac{-4}{2(2)} = -1$$

$$y = 2(-1)^2 + 4(-1) - 2$$

$$y = -4 \quad \boxed{\text{vertex: } (-1,-4)}$$

$$y = 2(x+1)^2 - 4$$

$$2 = \frac{1}{4c}$$

$$8c = 1$$

$$c = \frac{1}{8} \rightarrow \boxed{\text{focus: } (-1, -3.875)}$$

directrix: $-4 - .875$

$\boxed{y = -4.875}$

★ A warm-up of simplifying radicals would be great for this day!

Day 5: Completing the Square & Quadratic Formula

Completing the Square is one way to solve quadratic equations when you don't want to or can't factor.

Example 1: Solve $x^2 + 6x - 7 = 0$ by completing the square.

Step 1: Move the constant to the other side.

$x^2+6x=7$

Step 2: Compute $\left(\frac{b}{2}\right)^2$ and add the result to both sides of the equation.

$\left(\frac{6}{2}\right)^2=9 \rightarrow x^2+6x+9=7+9$

Step 3: Convert the left side to a binomial squared and simplify the right side.

$(x+3)^2=16$

Step 4: Square root both sides, and don't forget the $\pm$ on the right side!

$\sqrt{(x+3)^2}=\sqrt{16} \rightarrow x+3=\pm 4$

Step 5: Solve for x. Remember that the $\pm$ gives you two solutions!

$x+3=4$ $\boxed{x=1}$

$x+3=-4$ $\boxed{x=-7}$

Example 2:

1) Solve $2a^2 + 12a + 10 = 0$ by completing the square.

$2a^2+12a=-10$

Factor out a 2 first!

$2(a^2+6a)=-10$

$\left(\frac{6}{2}\right)^2=9$

$2(a^2+6a+9)=-10+18$

$2(a+3)^2=8$

$(a+3)^2=4$

$a+3=\pm 2$

$a+3=2$ $\boxed{a=-1}$

$a+3=-2$ $\boxed{a=-5}$

You try! Solve the following by completing the square. (It's okay to get decimals!)

a) $n^2 + 13n + 22 = 7$

$n^2+13n=-15$

$\left(\frac{13}{2}\right)^2=42.25$

$n^2+13n+42.25=-15+42.25$

$(n+6.5)^2=27.25$

$n+6.5=\pm 5.22$

$n+6.5=5.22$ $\boxed{n=-1.28}$

$n+6.5=-5.22$ $\boxed{n=-11.72}$

b) $4v^2 + 16v = 65$

$4(v^2+4v)=65$

$\left(\frac{4}{2}\right)^2=4$

$4(v^2+4v+4)=65+16$

$4(v+2)^2=81$

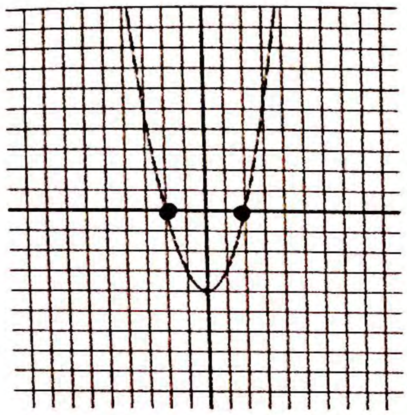
$(v+2)^2=20.25$

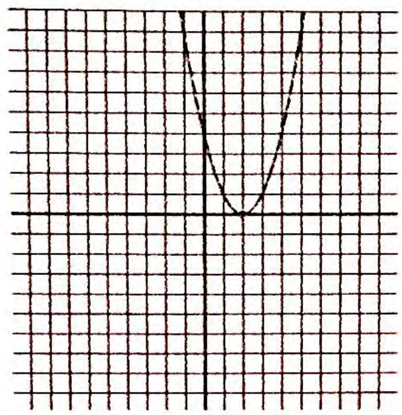
$v+2=\pm 4.5$

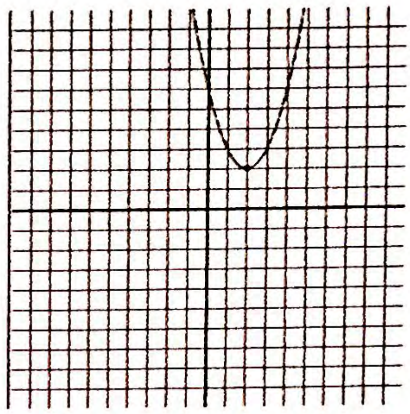
$v+2=4.5$ $\boxed{v=2.5}$

$v+2=-4.5$ $\boxed{v=-6.5}$

The Quadratic Formula







How many solutions? two
Type: Real, rational
Discriminant is: positive

How many solutions? one
Type: real, rational
Discriminant is: 0

How many solutions? two
Type: imaginary
Discriminant is: negative

The quadratic formula is a method for solving quadratic equations that works every time! The solutions are the parabola's zeros.

What is the quadratic formula? Circle the discriminant!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

discriminant (doesn't include the square root!)

Example 1: Use the discriminant to find the number and types of solutions to the quadratic expression. Remember to get all terms on one side and in standard form!

a) $3x^2 - 5x - 18 = 0$

$b^2 - 4ac$

$(-5)^2 - 4(3)(-18) = \boxed{241}$

$\boxed{2\text{ real}}$

b) $4x^2 + 5 = 2x$

$4x^2 - 2x + 5 = 0$

$(-2)^2 - 4(4)(5)$

$\boxed{= -76}$ $\boxed{2\text{ imag.}}$

c) $2x^2 = 3x - 12$

$2x^2 - 3x + 12 = 0$

$(-3)^2 - 4(2)(12)$

$\boxed{= -87}$ $\boxed{2\text{ imag.}}$

Example 2: Use the quadratic formula to solve $3x^2 - 5x = 2$. Then, sketch the graph using what you know about the vertex and parabolic transformations.

a 3
b -5
c -2

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)} = \frac{5 \pm \sqrt{49}}{6}$$

$$= \frac{5 \pm 7}{6}$$

$$= \frac{5+7}{6} \qquad \frac{5-7}{6}$$

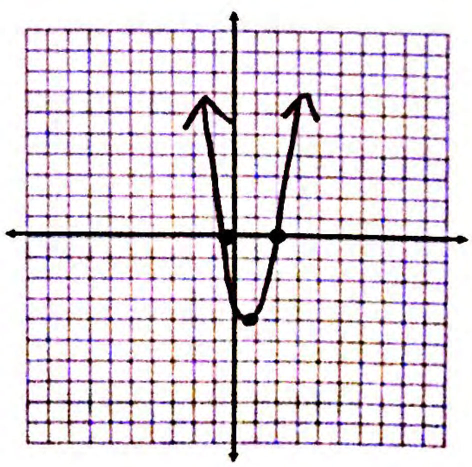
$\boxed{x = 2}$ $\boxed{x = -\frac{1}{3}}$

vertex: $x = \frac{5}{2(3)} = \frac{5}{6}$

$y = 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) - 2$

$y = -\frac{49}{12}$

vertex: $\left(\frac{5}{6}, -\frac{49}{12}\right)$



I would recommend doing this one with your kids to help them w/ simplifying radicals.

You try! Use the quadratic formula to solve $5x^2 + 8x - 11 = 0$. Then, sketch the graph using what you know about the vertex and parabolic transformations.

a $\underline{5}$
b $\underline{8}$
c $\underline{-11}$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(-11)}}{2(5)} = \frac{-8 \pm \sqrt{284}}{10}$

$= \frac{-8 \pm \sqrt{4} \cdot \sqrt{71}}{10}$

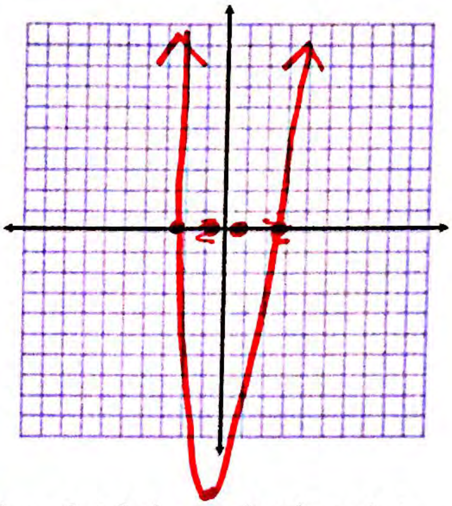
$= \frac{-8 \pm 2\sqrt{71}}{10}$

$= -\frac{8}{10} \pm \frac{2\sqrt{71}}{10} = \boxed{-\frac{4}{5} \pm \frac{\sqrt{71}}{5}}$

vertex: $x = \frac{-8}{2(5)} = \frac{-8}{10} = \frac{-4}{5}$

$y = 5(-\frac{4}{5})^2 + 8(-\frac{4}{5}) - 11$

$= -\frac{71}{5}$ $\boxed{(-\frac{4}{5}, -\frac{71}{5})}$



You try! Use the quadratic formula to solve $9x^2 - 11 = 6x$. Then, sketch the graph using what you know about the vertex and parabolic transformations.

$9x^2 - 6x - 11 = 0$

a $\underline{9}$
b $\underline{-6}$
c $\underline{-11}$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(9)(-11)}}{2(9)} = \frac{6 \pm \sqrt{432}}{18}$

$= \frac{6 \pm \sqrt{144} \cdot \sqrt{3}}{18}$

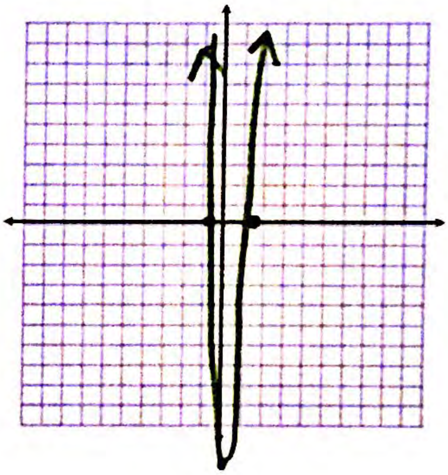
$= \frac{6 \pm 12\sqrt{3}}{18} = \frac{6}{18} \pm \frac{12\sqrt{3}}{18}$

$\boxed{x = \frac{1}{3} \pm \frac{2\sqrt{3}}{3}}$

vertex: $x = \frac{6}{2(9)} = \frac{6}{18} = \frac{1}{3}$

$y = 9(\frac{1}{3})^2 - 6(\frac{1}{3}) - 11$

$= \boxed{(\frac{1}{3}, -12)}$



Modeling Using Quadratics

1) Each year, Heritage's Homecoming committee organizes a dance. Based on previous years, the organizers decided that the Income from ticket sales, I(t) is related to ticket price *t* by the equation $I(t) = 400t - 40t^2$.

What ticket price(s) would generate the greatest income? What is the greatest income possible?

Ticket price(s) $5

Income $1,000

Use your calculator to find the maximum!

2) The equation $h = -16t^2 + 40t$ describes the height h, in feet, of a ball that is thrown straight up as a function of the time t, in seconds, that the ball has been in the air.

Use calculator!

At what time does the ball reach its maximum height? 1.25 seconds

What is the maximum height? 25 ft.

When does the ball hit the ground? 2.5 seconds

Day 6: Complex Number Operations

You already know about real numbers (rational and irrational), but there are also imaginary numbers that use the letter i.

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -\sqrt{-1}$

$i^4 = 1$

Simplifying Using i

Example 1: Simplify $\sqrt{-8}$.

$\sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{2}$

$= i \cdot \pm 2 \cdot \sqrt{2}$

$\boxed{= \pm 2i\sqrt{2}}$

You try! Simplify the following:

a) $\sqrt{-12}$

$\sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{3}$

$= i \cdot \pm 2 \cdot \sqrt{3}$

$\boxed{= \pm 2i\sqrt{3}}$

b) $\sqrt{-13}$

$\sqrt{-1} \cdot \sqrt{13}$

$\boxed{= i\sqrt{13}}$

Simplifying Complex Numbers

Standard Form of a Complex Number: $a+bi$

Example 2: Write $\sqrt{-9} + 6$ in standard form.

$\sqrt{-1} \cdot \sqrt{9} + 6$

$= i \cdot \pm 3 + 6$

$= \pm 3i + 6 = \boxed{6 \pm 3i}$

You try! Write $-\sqrt{-50} - 2$ in standard form.

$-1 \cdot \sqrt{-1} \cdot \sqrt{25} \cdot \sqrt{2} - 2$

$i^3 \cdot \pm 5 \cdot \sqrt{2} - 2$

$\pm 5i^3\sqrt{2} - 2$

$= \boxed{-2 \pm 5i^3\sqrt{2}}$

Adding and Subtracting

Example 3: Simplify $(5 + 7i) + (-2 + 6i)$

$5 + 7i - 2 + 6i$

$= \boxed{3 + 13i}$

You try! a) Simplify $(-4 + 6i) + (3 - 2i)$

$-4 + 6i + 3 - 2i$

$\boxed{-1 + 4i}$

b) Simplify $(8 + 3i) - (2 + 4i)$

$8 + 3i - 2 - 4i$

$\boxed{6 - i}$

Multiplying Complex Numbers

Example 4: Simplify $(5i)(-4i)$.
$= -20i^2$
$= -20(-1)$
$\boxed{= 20}$

You try! Simplify $(12i)(7i)$.
$= 84i^2$
$= 84(-1) = \boxed{-84}$

Example 5: Simplify $(2+3i)(-3+5i)$.

FOIL!
$-6+10i-9i+15i^2$
$-6+i+15(-1)$
$-6+i-15 = \boxed{-21+i}$

F: Firsts
O: Outsides
I: Insides
L: Lasts

You try! a) Simplify $(6-5i)(4-3i)$.
$24-18i-20i+15i^2$
$24-38i+15(-1)$
$24-38i-15$
$\boxed{9-38i}$

b) Simplify $3i(9-4i)$.
$27i-12i^2$
$27i-12(-1)$
$27i+12$
$\boxed{12+27i}$

c) Simplify $(8-2i)^2$.
$(8-2i)(8-2i)$
$64-16i-16i+4i^2$
$64-32i-4$
$\boxed{60-32i}$

Rationalizing

There is one big rule for complex number, and that is that you cannot have i in the denominator!

Why do you think this is? Because you can't have square roots in the denominator, and $i=\sqrt{-1}$.

Why do you think we call it rationalizing? Because it makes the denominator rational!

Rationalizing with One Term in the Denominator

Example 6: Simplify $\frac{3+8i}{5i} \cdot \frac{i}{i} = \frac{3i+8i^2}{5i^2}$
$= \frac{3i-8}{-5} = \boxed{\frac{-8+3i}{-5}} = \boxed{\frac{8}{5} + \frac{3i}{-5}}$

You try! Simplify $\frac{4-2i}{-6i} \cdot \frac{i}{i} = \frac{4i-2i^2}{-6i^2}$
$= \frac{4i+2}{6} = \frac{4i}{6} + \frac{2}{6} = \boxed{\frac{1}{3} + \frac{2i}{3}}$

Rationalizing with a Binomial in the Denominator

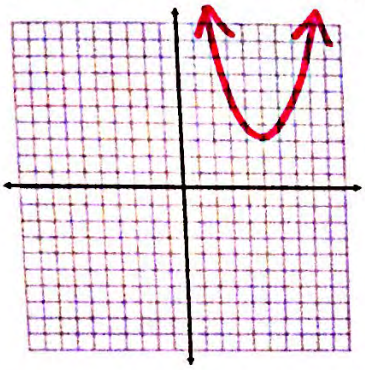
Example 7: Simplify $\frac{4i}{6+2i} \cdot \frac{6-2i}{6-2i}$
$\frac{24i-8i^2}{36-\cancel{12i}+\cancel{12i}-4i^2} = \frac{24i+8}{36+4}$
$= \frac{8+24i}{40} = \frac{8}{40} + \frac{24i}{40}$
$= \boxed{\frac{1}{5} + \frac{3i}{5}}$

You try! Simplify $\frac{(2+3i)}{(5-2i)} \cdot \frac{5+2i}{5+2i}$
$\frac{10+4i+15i+6i^2}{25+\cancel{10i}-\cancel{10i}-4i^2}$
$= \frac{10+19i-6}{25+4} = \frac{4+19i}{29}$
$= \boxed{\frac{4}{29} + \frac{19i}{29}}$

What is a **complex conjugate?** The same binomial with the opposite sign (ex. $(4+3i$ and $4-3i)$
Why do we use it? Because it eliminates the i term in the denominator.

Day 7: Finding Complex/Imaginary Solutions

Quick review! Sketch the type of parabola that would have complex/imaginary roots.



Why does this parabola have imaginary roots?

Because it never actually crosses the x-axis! It has no zeros.

Let's solve some quadratic equations that have complex solutions!

Example 1: Solve $4x^2 + 100 = 0$.

$4x^2 = -100$

$x^2 = -25$

$x = \pm\sqrt{-25}$

$x = \pm\sqrt{-1}\cdot\sqrt{25}$

$x = \pm i \cdot 5$

$\boxed{x = \pm 5i}$

You try!

a) Simplify $3x^2 + 48 = 0$

$3x^2 = -48$

$x^2 = -16$

$x = \pm\sqrt{-16}$

$x = \pm\sqrt{-1}\cdot\sqrt{16}$

$x = \pm i \cdot 4$

$\boxed{x = \pm 4i}$

b) Simplify $-5x^2 - 150 = 0$

$-5x^2 = 150$

$x^2 = -30$

$x = \pm\sqrt{-30}$

$x = \pm\sqrt{-1}\cdot\sqrt{30}$

$\boxed{x = \pm i\sqrt{30}}$

Example 2: Solve $2x^2 = -6x - 7$

$2x^2 + 6x + 7 = 0$

$x = \dfrac{-6 \pm \sqrt{(6)^2 - 4(2)(7)}}{2(2)}$

$x = \dfrac{-6 \pm \sqrt{-20}}{4}$

$x = \dfrac{-6 \pm \sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{5}}{4}$

$x = \dfrac{-6 \pm i \cdot 2 \cdot \sqrt{5}}{4}$

$x = \dfrac{-6 \pm 2i\sqrt{5}}{4} = \dfrac{-6}{4} \pm \dfrac{2i\sqrt{5}}{4} = \boxed{-\dfrac{3}{2} \pm \dfrac{i\sqrt{5}}{2}}$

You try! Solve $6x^2 - 3x + 2 = 0$

$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(6)(2)}}{2(6)}$

$x = \dfrac{3 \pm \sqrt{-39}}{12}$

$x = \dfrac{3 \pm \sqrt{-1}\cdot\sqrt{39}}{12}$

$x = \dfrac{3 \pm i\sqrt{39}}{12}$

$x = \dfrac{3}{12} \pm \dfrac{i\sqrt{39}}{12}$

$\boxed{x = \dfrac{1}{4} \pm \dfrac{i\sqrt{39}}{12}}$

Quadratic Equation	Value of Discriminant (show work!)	Number of Solutions (or roots)	Types of Solutions (or roots)	Using the quadratic formula, what are the roots/solutions/zeros? (show work!)
$-3p^2-8p+4=10$ $-3p^2-8p-6=0$	b^2-4ac $=(-8)^2-4(-3)(-6)$ $=\boxed{-8}$	2 imag.	imaginary	$x=\frac{8\pm\sqrt{-8}}{2(-3)}$ $x=\frac{8\pm\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{2}}{-6}$ $x=\frac{8\pm2i\sqrt{2}}{-6}=\frac{8}{-6}\pm\frac{2i\sqrt{2}}{-6}$ $=\boxed{-\frac{4}{3}\pm\frac{i\sqrt{2}}{-3}}$
$-4n^2+4n=1$ $-4n^2+4n-1=0$	b^2-4ac $=(4)^2-4(-4)(-1)$ $=\boxed{0}$	1	real	$x=\frac{-4\pm\sqrt{0}}{2(-4)}$ $x=\frac{-4}{-8}=\boxed{\frac{1}{2}}$

Sum and Difference of Cubes

$a^3+b^3=(a+b)(a^2-ab+b^2)$

same

opposite

always +

$a^3-b^3=(a-b)(a^2+ab+b^2)$

same

opposite

always +

Example 3: Factor and solve $x^3-8=0$ using Difference of Cubes.

$a=x$
$b=2$

$(x-2)(x^2+2x+4)=0$

$(x-2)=0$

$\boxed{x=2}$

$x=\frac{-2\pm\sqrt{(2)^2-4(1)(4)}}{2(1)}$

$x=\frac{-2\pm\sqrt{-12}}{2}$

$x=\frac{-2\pm\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{3}}{2}$

$x=\frac{-2\pm2i\sqrt{3}}{2}$

$x=\frac{-2}{2}\pm\frac{2i\sqrt{3}}{2}$

$\boxed{x=-1\pm i\sqrt{3}}$

1 real solution, 2 imag. solutions

You try!

a) $27x^3-1=0$

$a=3x$
$b=1$

$(3x-1)(9x^2+3x+1)=0$

$3x-1=0$

$3x=1$

$\boxed{x=\frac{1}{3}}$

$x=\frac{-3\pm\sqrt{(3)^2-4(9)(1)}}{2(9)}$

$x=\frac{-3\pm\sqrt{-27}}{18}$

$x=\frac{-3\pm3i\sqrt{3}}{18}=\boxed{-\frac{1}{6}\pm\frac{i\sqrt{3}}{6}}$

b) $24x^3+192=0$

$a=x$
$b=2$

$24(x^3+8)=0$

$x^3+8=0$

$(x+2)(x^2-2x+4)=0$

$x+2=0$

$\boxed{x=-2}$

$x=\frac{2\pm\sqrt{(-2)^2-4(1)(4)}}{2(1)}$

$x=\frac{2\pm\sqrt{-12}}{2}$

$x=\frac{2\pm2i\sqrt{3}}{2}=\boxed{1\pm i\sqrt{3}}$

Factoring by Substitution

Example 4: Factor and solve $x^4 - 2x^2 - 8 = 0$ $z = x^2$

$z^2 - 2z - 8 = 0$ $-4, 2$

$(z^2 - 4z) + (2z - 8) = 0$

$z(z-4) + 2(z-4) = 0$

$(z+2)(z-4) = 0$

$(x^2+2)(x^2-4) = 0$

$(x^2+2)(x+2)(x-2) = 0$

$x^2 + 2 = 0$, $x^2 = -2$, $x = \pm\sqrt{-2}$, $\boxed{x = \pm i\sqrt{2}}$ | $\boxed{x=-2}$ | $\boxed{x=2}$

You try! a) $n^4 + 4n^2 - 12 = 0$ $z = n^2$

$z^2 + 4z - 12 = 0$ $6, -2$

$(z^2 + 6z)(-2z - 12) = 0$

$z(z+6) - 2(z+6) = 0$

$(z-2)(z+6) = 0$

$(x^2-2)(x^2+6) = 0$

$x^2 - 2 = 0$, $x^2 = 2$, $\boxed{x = \pm\sqrt{2}}$ | $x^2 + 6 = 0$, $x^2 = -6$, $x = \pm\sqrt{-6}$, $\boxed{x = \pm i\sqrt{6}}$

b) $3w^4 - 8w^2 + 4 = 0$ $z = w^2$

$3z^2 - 8z + 4 = 0$ 12 $-6, -2$

$(3z^2 - 6z)(-2z + 4) = 0$

$3z(z-2) - 2(z-2) = 0$

$(3z-2)(z-2) = 0$

$(3x^2-2)(x^2-2) = 0$

$3x^2 - 2 = 0$, $3x^2 = 2$, $x^2 = \frac{2}{3}$, $\boxed{x = \pm\sqrt{\frac{2}{3}}}$ | $x^2 - 2 = 0$, $x^2 = 2$, $\boxed{x = \pm\sqrt{2}}$

Function	*# Of Zeros (1 pt)*	*# Of Real Zeros (1 pt)*	*List of All Zeros (Exact – no decimals) (2 pts)*
$f(x) = x^3 - 10x^2 + 13x$	3	3	$x = 0, 5 \pm 2\sqrt{3}$
$f(x) = x^4 - x^2 - 12$	4	2	$x = \pm i\sqrt{3}, -2, 2$
$f(x) = x^3 + 27$	3	1	$x = -3, \frac{3}{2} \pm \frac{3i\sqrt{3}}{2}$

$x(x^2 - 10x + 13) = 0$

$\boxed{x = 0}$

$x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(13)}}{2(1)}$

$x = \frac{10 \pm \sqrt{100 - 52}}{2}$

$x = \frac{10 \pm \sqrt{48}}{2}$

$x = \frac{10 \pm \sqrt{16}\cdot\sqrt{3}}{2}$

$x = \frac{10 \pm 4\sqrt{3}}{2} = \boxed{5 \pm 2\sqrt{3}}$

$z = x^2$ -12

$z^2 - z - 12 = 0$ $-4, 3$

$(z^2 - 4z)(3z - 12) = 0$

$z(z-4) + 3(z-4) = 0$

$(z+3)(z-4) = 0$

$(x^2+3)(x^2-4) = 0$

$(x^2+3)(x+2)(x-2) = 0$

$x^2 + 3 = 0$, $x^2 = -3$, $\boxed{x = \pm i\sqrt{3}}$ | $x + 2 = 0$, $\boxed{x = -2}$ | $x - 2 = 0$, $\boxed{x = 2}$

$a = x$

$b = 3$

$(x+3)(x^2 - 3x + 9) = 0$

$x + 3 = 0$, $\boxed{x = -3}$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$

$x = \frac{3 \pm \sqrt{-27}}{2}$

$x = \frac{3 \pm \sqrt{-1}\cdot\sqrt{9}\cdot\sqrt{3}}{2}$

$x = \frac{3 \pm 3i\sqrt{3}}{2} = \boxed{\frac{3}{2} \pm \frac{3i\sqrt{3}}{2}}$