Math III Unit 3 Part 2: POLYNOMIAL MODELING AND EQUATIONS

Lauren Winstead, Heritage High School

Main topics of instruction:

1) Polynomial Degree and End Behavior
2) Adding and Subtracting Polynomials, Degree, and Zeros
3) Multiplying and Dividing Polynomials
4) Intersections of two graphs as $f(x) = g(x)$

Day 1: Polynomial Degree and End Behavior

Standard Form of a Polynomial: $P(x) = ax^3 + bx^2 + cx + d$

Degree: The highest exponent in a standard form polynomial.

$$P(x) = 2x^3 - 5x^2 - 2x + 5$$

Degree	Name Using Degree	Polynomial Example	Number of Terms	Name Using Number of Terms
0	constant	$P(x) = 4$	1	monomial
1	linear	$P(x) = 3x + 2$	2	binomial
2	quadratic	$P(x) = 2x^2 - 6x + 3$	3	trinomial
3	cubic	$P(x) = 5x^3 - 2x^2 + 5x - 1$	4	polynomial
4	quartic	$P(x) = 3x^4 - x^3 + x^2 - 4x + 1$	5	polynomial
5	quintic	$P(x) = 2x^5 - 4x^4 + 3x^3 - 2x^2 + 5x + 1$	6	polynomial

You try! Write the following polynomial in standard form and classify by degree and number of terms.

$$P(x) = 3x^3 - 32x^2 + 48x + x^3 \qquad P(x) = 4x^3 - 32x^2 + 48x$$

Degree: 3 Name Using Degree: cubic Name Using Number of Terms: trinomial

Sometimes, the polynomial is in factored form and looks like this:

$$y = x^2(-2x + 12)(-2x + 4)^3$$

Degree: 6 Name Using Degree: hexic Name Using Number of Terms: polynomial

Multiply it out!

$y = (-2x^3 + 12x^2)(-2x+4)(-2x+4)(-2x+4)$

$y = 4x^4 - 8x^3 - 24x^3 + 48x^2 (-2x+4)(-2x+4)$

$y = 4x^4 - 32x^3 + 48x^2 (-2x+4)(-2x+4)$

$y = -8x^5 + 16x^4 + 64x^4 - 128x^3 - 96x^3 + 192x^2 (-2x+4)$

definitely a polynomial! $\rightarrow$ $y = -8x^5 + 80x^4 - 224x^3 + 192x^2(-2x+4)$

Domain: The set of all possible x values for a function

Range: The set of all possible y-values

What are the domain and range of $P(x) = 3x^3 - 32x^2 + 48x + x^3$? Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Let's say that this polynomial represents a football player's average speed over the amount of time he has been at practice. What is the practical domain for the function now? What is the practical range? Domain: $[0, 2]$
Range: $[0, 20.2]$

Comparing Models

Example 1: For the following set of points, which type of model fits best? A linear, quadratic, or cubic model? Make sure you turn your diagnostics on!

x	0	5	10	15	20
y	10.1	2.8	8.1	16.0	17.8

Linear: $y = .572x + 5.24$
$r^2 = .55$
Quad.: $y = .06x^2 - .62x + 8$
$r^2 = .76$
$\boxed{\text{Cubic:}}$ $y = -.01x^3 + .43x^2 - 3$
$+10.0$
$r^2 = .99$

You try! Which model fits the data best?

near: $y = -.01x + 14.4$ $r^2 = .0002$

x	1	7	13	19	23
y	11.2	14.3	21.1	15.2	9.8

uad: $y = -.07x^2 + 1.72x + 8.56$
$r^2 = .82$

$\boxed{\text{bic}}$ $y = -.004x^3 + .07x^2 + .41x + 10.43$ $r^2 = .90$

Investigating End Behavior for Polynomial Functions

Graph each function on your calculator. Use your graph to fill in the chart.

Graph	Is the degree even or odd?	Is the leading coefficient positive or negative?	Does the graph rise or fall on the left?	Does the graph rise or fall on the right?
1. $y = x^2$	even	positive	rise	rise
2. $y = x^4$	even	positive	rise	rise
3. $y = -x^2$	even	negative	fall	fall
4. $y = -x^4$	even	negative	fall	fall
5. $y = x^3$	odd	positive	fall	rise
6. $y = x^5$	odd	positive	fall	rise
7. $y = -x^3$	odd	negative	rise	fall
8. $y = -x^5$	odd	negative	rise	fall

End Behavior of a Polynomial Function				
	Leading coefficient is **Positive**		Leading coefficient is **Negative**	
	Left	Right	Left	Right
Degree is **odd**	fall	~~positive~~ rise	rise	fall
Degree is **even**	rise	rise	fall	fall

SUMMARY: The end behavior of a polynomial depends on:

1. Whether the degree of the polynomial is even or odd.
2. The sign of the leading coefficient.

You Try!!

Equation	Degree	Leading coefficient	End Behavior
1. $y = x^3 - 3x^2 - 3x + 9$	odd	positive	fall / rise
2. $y = -2x^4 + 7x^2 - 6$	even	negative	fall / fall
3. $y = -3x^3 + x^2 + 10x - 5$	odd	negative	rise / fall
4. $y = 8x^4 + 11x^3 + 5x^2$	even	positive	rise / rise
5. $y = -2x^4 - 5$	even	negative	fall / fall

END BEHAVIOR----HAND BEHAVIOR

Degree	Leading Coefficient	Left-hand Behavior ($x \to -\infty$)	Right-hand Behavior ($x \to \infty$)	Picture
Even	Positive	$y \to \infty$	$y \to \infty$	
Even	Negative	$y \to -\infty$	$y \to -\infty$	
Odd	Positive	$y \to -\infty$	$y \to \infty$	
Odd	Negative	$y \to \infty$	$y \to -\infty$	

Multiplicity

What is multiplicity? The number of times a solution appears in a polynomial (or, the exponent outside the binomial

Example 2: In each of the following factored polynomials, what is the multiplicity of each zero?

a) $y = (x - 1)(x + 2)^2(x - 3)^3$

$x - 1 = 0$, $\boxed{x = 1}$, mult. $= 1$

$x + 2 = 0$, $\boxed{x = -2}$, mult. $= 2$

$x - 3 = 0$, $\boxed{x = 3}$, mult. $= 3$

b) $x(2x - 4)^3(3x + 1)^2$

$\boxed{x = 0}$, mult. $= 1$

$2x - 4 = 0$, $2x = 4$, $\boxed{x = 2}$, mult. $= 3$

$3x + 1 = 0$, $3x = -1$, $x = -\frac{1}{3}$, mult. $= 2$

c) $y = (5x - 4)^2(3x - 6)$

$5x - 4 = 0$, $5x = 4$, $\boxed{x = \frac{4}{5}}$, mult. $= 2$

$3x - 6 = 0$, $3x = 6$, $\boxed{x = 2}$, mult. $= 1$

Rules of Multiplicity:

- A multiplicity of 1 will go straight through the x-axis
- A multiplicity of 2 will bounce off the x-axis
- A multiplicity of 3 will curve through the x-axis

Using what you know about end behavior and multiplicity, sketch each of the polynomials from Example 2.

LC: pos.
Deg: 6
~~(fall/rise)~~
(rise/rise)

a) $y = (x-1)(x+2)^2(x-3)^3$

$x-1=0$	$x+2=0$	$x-3=0$
$x=1$	$x=-2$	$x=3$
mult. $=1$	mult. $=2$	mult. $=3$

LC: pos. Deg.: 6 (rise/rise)

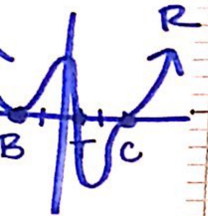
b) $x(2x-4)^3(3x+1)^2$

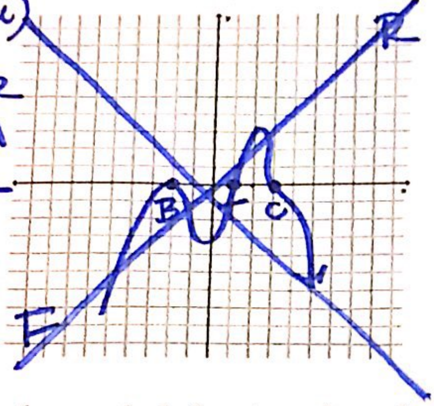
$\boxed{x=0}$	$2x-4=0$	$3x+1=0$
mult. $=1$	$2x=4$	$3x=-1$
	$\boxed{x=2}$	$\boxed{x=-\frac{1}{3}}$
	mult. $=3$	mult. $=2$

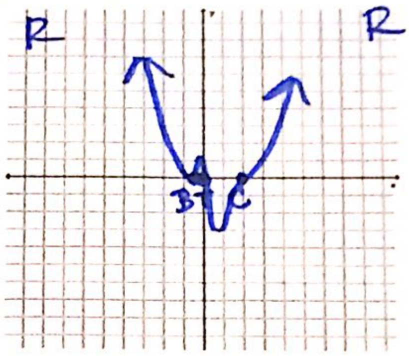
LC: pos. Deg.: 3 (fall/rise)

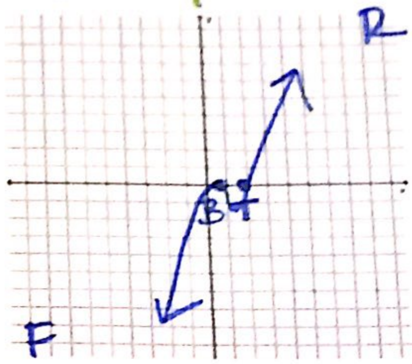
c) $y = (5x-4)^2(3x-6)$

$5x-4=0$	$3x-6=0$
$5x=4$	$3x=6$
$\boxed{x=4/5}$	$\boxed{x=2}$
mult. $=2$	mult. $=1$



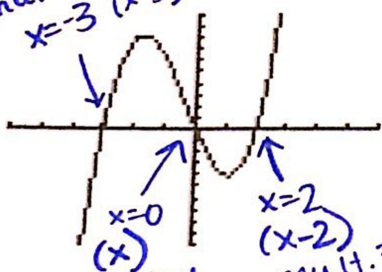






Use the graphs below to write polynomials that would fit the graphs, based on their zeros, end behavior, and multiplicity.

mult. $=1$
$x=-3$ $(x+3)$



$x=0$
(x)
mult. $=1$

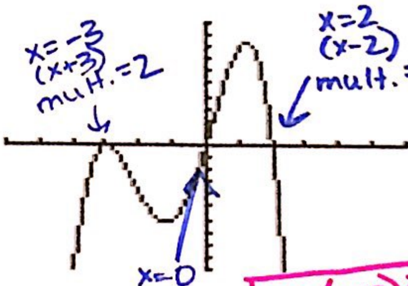
$x=2$
$(x-2)$
mult. $=1$

$\boxed{P(x)=(x+3)(x)(x-2)}$

Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$

$x=-3$
$(x+3)$
mult. $=2$

$x=2$
$(x-2)$
mult. $=1$

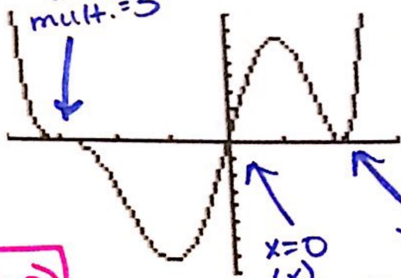


$x=0$
(x)
mult. $=1$

$\boxed{y=(x+3)^2(x)(x-2)}$

Domain: $(-\infty,\infty)$
Range: $(-\infty,8]$

$x=-3$
$(x+3)$
mult. $=3$



$x=0$
(x)
mult. $=1$

$x=$
$(x$

$\boxed{y=(x+3)^3(x)(x-2)^2}$

Domain: $(-\infty,\infty)$
Range: $[-7,\infty)$

Suppose oceanographers are using the first graph to study the days of the month when the temperature of the ocean is below zero. What is the practical domain? $(0,2)$ What is the practical range? $[-4,0)$

Day 2: Adding and Subtracting Polynomials, Degree, and Roots

Adding and subtracting polynomials is all about combining like terms.

Example 1: Simplify $(-4k^4 + 14 + 3k^2) + (-3k^4 - 14k^2 - 8)$. Put answer in standard form.

$-4k^4 - 3k^4 + 14 - 8 + 3k^2 - 14k^2$

$-7k^4 + 6 - 11k^2 \quad = \boxed{-7k^4 - 11k^2 + 6}$

You try! Simplify $(9r^3 + 5r^2 + 11r) + (-2r^3 + 9r - 8r^2)$. Put answer in standard form.

$9r^3 + 5r^2 + 11r - 2r^3 + 9r - 8r^2$

$\boxed{7r^3 - 3r^2 + 20r}$

Example 2: Simplify $(12a^5 - 6a - 10a^3) - (10a - 2a^5 - 14a^4)$. Put answer in standard form.

$12a^5 - 6a - 10a^3 - 10a + 2a^5 + 14a^4$

$14a^5 - 16a - 10a^3 + 14a^4$

$\boxed{14a^5 + 14a^4 - 10a^3 - 16a}$

You try! Simplify $(8n - 3n^4 + 10n^2) - (3n^2 + 11n^4 - 7)$. Put answer in standard form.

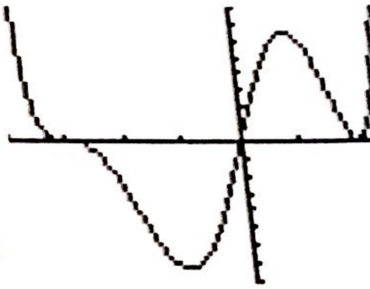
$8n - 3n^4 + 10n^2 - 3n^2 - 11n^4 + 7$

$8n - 14n^4 + 7n^2 + 7$

$\boxed{-14n^4 + 7n^2 + 8n + 7}$

Writing Polynomial Equations

Yesterday, you wrote polynomial functions from graphs, like this one:



Roots: x = -3 with multiplicity 3
x = 0 with multiplicity 1
x = 2 with multiplicity 2

$$y = (x+3)^3(x-0)(x-2)^2$$

Could you do it without a graph?

Example 3: Write a polynomial function **in standard form** with roots at x = 4, multiplicity 2, x = -2, multiplicity 1, and x = ½ , multiplicity 3.

$(x-4)^2$

$(x+2)$ $\quad (2x-1)^3$

$\boxed{y = (x-4)^2(x+2)(2x-1)^3}$

You try! Write a polynomial function **in standard form** with roots at x = -8, multiplicity 1, x = ¾, multiplicity 3, and x = 0, multiplicity 1.

$(x+8)^1$

$(4x-3)^3$ $\quad (x)^1$

$\boxed{y = (x+8)^1(4x-3)^3(x)}$

Imaginary and Irrational Root Theorems

Rule 1: If a polynomial has a root of $\sqrt{x}$, it also has a root of $-\sqrt{x}$.

Rule 2: If a polynomial has a root of $a + \sqrt{x}$, it also has a root at $a - \sqrt{x}$.

Rule 3: If a polynomial has a root of bi, it also has a root of $-bi$.

Rule 4: If a polynomial has a root of $a + bi$, it also has a root of $a - bi$.

☼ ***Does not apply to real rational roots.***

Example 4: A polynomial has roots at $4 - \sqrt{6}$ and $\sqrt{3}$. What are the two other roots?

$4+\sqrt{6}$ and $-\sqrt{3}$

You try! A polynomial has roots at 2 and 4 + i. What are the two other roots?

Trick question! There is only one other at 4-i.

Example 5: Find a 3[rd] degree polynomial equation with rational coefficients that has -5 and 1 – i as roots.

$y = (x+5)(x-(1-i))(x-(1+i))$

$y = (x+5)(x-1+i)(x-1-i)$

$y = (x+5)(x^2 - x \cancel{-ix} - x + 1 \cancel{+i} \cancel{+ix} \cancel{-i} - i^2)$

$y = (x+5)(x^2-2x+1-(-1))$

$y = (x+5)(x^2-2x+2)$

$y = (x^3-2x^2+2x+5x^2-10x+10) = \boxed{(x^3+3x^2-8x+1}$

You try! Find a 3[rd] degree polynomial equation with rational coefficients that has roots at -2 and 5i.

$y = (x+2)(x-5i)(x+5i)$

$y = (x+2)(x^2+\cancel{5ix}\cancel{-5ix}-25i^2)$

$y = (x+2)(x^2+25)$

$y = x^3+25x+2x^2+50$

$\boxed{y = x^3+2x^2+25x+50}$

Day 3: Factoring, Multiplying, and Dividing Polynomials

Sum and Difference of Cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

same sign — opposite sign — always +

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

same sign — opposite sign — always +

Example 3: Factor and solve $x^3 - 8 = 0$ using Difference of Cubes.

$a = \sqrt[3]{x^3} = x$

$b = \sqrt[3]{8} = 2$

$(x-2)(x^2+2x+4)$

$x-2=0$

$\boxed{x=2}$

$x = \frac{-2 \pm \sqrt{(2)^2-4(1)(4)}}{2(1)} = \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm \sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{3}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2} = \boxed{-1 \pm i\sqrt{3}}$

You try! a) $27x^3 - 1 = 0$

$a = \sqrt[3]{27x^3} = 3x$

$b = \sqrt[3]{1} = 1$

$(3x-1)(9x^2+3x+1)$

$3x-1=0$

$3x=1$

$\boxed{x=\frac{1}{3}}$

$x = \frac{-3 \pm \sqrt{(3)^2-4(9)(1)}}{2(9)}$

$x = \frac{-3 \pm \sqrt{-27}}{18} = \frac{-3 \pm \sqrt{-1}\cdot\sqrt{9}\cdot\sqrt{3}}{18} = \frac{-3 \pm 3i\sqrt{3}}{18} = \boxed{\frac{-1}{6} \pm \frac{i\sqrt{3}}{6}}$

b) $24x^3 + 192 = 0$

$24(x^3+8)=0$

$(x^3+8)=0$

$a = x$

$b = 2$

$(x+2)(x^2+2x+4)$

$x+2=0$

$\boxed{x=-2}$

$x = \frac{-2 \pm \sqrt{(2)^2-4(1)(4)}}{2(1)} = \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2} = \boxed{-1 \pm i\sqrt{3}}$

Factoring by Substitution

(Note: You can also use normal trinomial factoring. Both are shown here →

Example 4: Factor and solve $x^4 - 2x^2 - 8 = 0$

Substitution $z = x^2$

$z^2 - 2z - 8 = 0$ (-8)

$(z^2 - 4z)(+2z - 8) = 0$

$z(z-4) + 2(z-4) = 0$

$(z+2)(z-4) = 0$

$(x^2+2)(x^2-4)=0$

$x^2+2=0$

$x^2=-2$

$x=\pm\sqrt{-2}$

$\boxed{x=\pm i\sqrt{2}}$

$x^2=4$

$\boxed{x=\pm 2}$

Normal Factoring

$x^4 - 2x^2 - 8 = 0$ (-8; $-4, 2$)

$(x^4 - 4x^2)(+2x^2 - 8) = 0$

$x^2(x^2-4) + 2(x^2-4) = 0$

$(x^2+2)(x^2-4) = 0$

$x^2+2=0$

$x^2=-2$

$\boxed{x=\pm i\sqrt{2}}$

$x^2-4=0$

$x^2=4$

$\boxed{x=\pm 2}$

You try! a) $n^4 + 4n^2 - 12 = 0$

$n^4 + 4n^2 - 12 = 0$ (-12; $6, -2$)

$(n^4 + 6n^2)(-2n^2 - 12) = 0$

$n^2(n^2+6) - 2(n^2+6) = 0$

$(n^2-2)(n^2+6) = 0$

$n^2-2=0$

$n^2=2$

$\boxed{n=\pm\sqrt{2}}$

$n^2+6=0$

$n^2=-6$

$n=\pm\sqrt{-6}$

$\boxed{n=\pm i\sqrt{6}}$

b) $3w^4 - 8w^2 + 4 = 0$

$3w^4 - 8w^2 + 4 = 0$ (12; $-6, -2$)

$(3w^4 - 6w^2)(-2w^2 + 4) = 0$

$3w^2(w^2-2) - 2(w^2-2) = 0$

$(3w^2-2)(w^2-2) = 0$

$3w^2-2=0$

$3w^2=2$

$w^2=\frac{2}{3}$

$\boxed{w=\pm\sqrt{\frac{2}{3}}}$

$w^2-2=0$

$w^2=2$

$\boxed{w=\pm\sqrt{2}}$

$x(x^2-10x+13)$

$\boxed{x=0}$ | $x=\frac{10\pm\sqrt{(-10)^2-4(1)(13)}}{2(1)}$

$x=\frac{10\pm\sqrt{48}}{2}$ $=\frac{10\pm\sqrt{16}\cdot\sqrt{3}}{2}$ $=\frac{10\pm4\sqrt{3}}{2}$ $=\boxed{5\pm2\sqrt{3}}$

$x^4-x^2-12=0$

$-4,3$

$(x^4-4x^2)+(3x^2-12)=0$

$x^2(x^2-4)+3(x^2-4)=0$

$(x^2+3)(x^2-4)=0$

$x^2+3=0$ | $x^2-4=0$

$x^2=-3$ | $x^2=4$

$x=\pm\sqrt{-3}$ | $\boxed{x=\pm2}$

$\boxed{x=\pm i\sqrt{3}}$

Function	*# Of Zeros (1 pt)*	*# Of Real Zeros (1 pt)*	*List of All Zeros (Exact – no decimals) (2 pts)*
$f(x)=x^3-10x^2+13x$	3	3	$x=0, 5\pm2\sqrt{3}$
$f(x)=x^4-x^2-12$	4	2	$x=\pm i\sqrt{3}, x=\pm2$
$f(x)=x^3+27$	3	1	$x=-3, \frac{3\pm3i\sqrt{3}}{2}$

$(x+3)(x^2-3x+9)=0$

$x+3=0$ | $x=\frac{3\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}=\frac{3\pm\sqrt{-27}}{2}=\frac{3\pm\sqrt{-1}\cdot\sqrt{9}\cdot\sqrt{3}}{2}=\boxed{\frac{3\pm3i\sqrt{3}}{2}}$

$\boxed{x=-3}$

Multiplying Polynomials

Example 5: Multiply and simplify $(3x-5)(-2x^2-3x-9)$.

$-6x^3-9x^2-27x+10x^3+15x+45$

$\boxed{4x^3-9x^2-12x+45}$

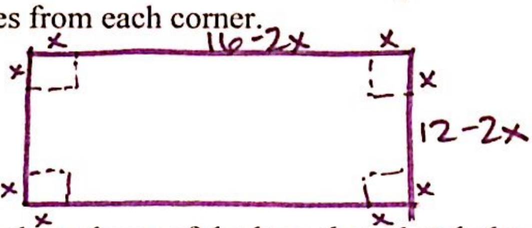
You try! Multiply and simplify 3x(x – 4)(x + 7).

$(3x^2-12x)(x+7)$

$3x^3+21x^2-12x^2-84x$ $=\boxed{3x^3+9x^2-84x}$

Let's apply! A metal worker wants to make an open box from a 12 in x 16 in sheet of metal by cutting equal squares from each corner.

Draw a picture:



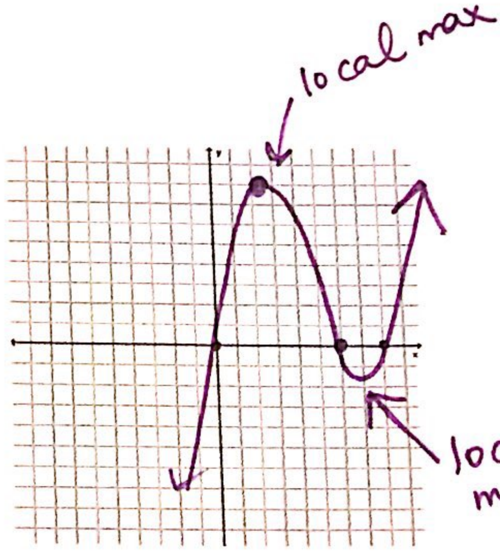
Write a function for the volume of the box, then sketch the graph.

$l=16-2x$

$w=12-2x$

$h=x$

$V=lwh=(16-2x)(12-2x)(x)$



Find the maximum volume of the box and the side length of the cut out squares that generates that volume.

Use 2nd → CALC → Max

(2.26, 194.07)

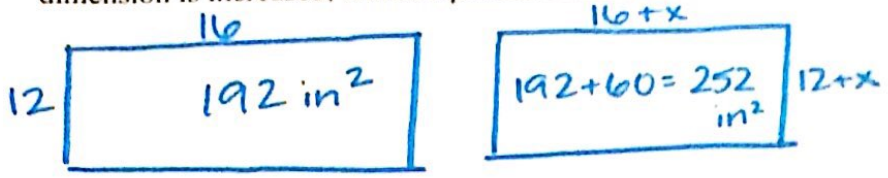
Max volume = 194.07 in³

cut out = 2.26 in.

You answer! What are the practical domain and practical range for this problem?

Domain: $(0,6)$ Range: $(0, 194.07]$

You try! A rectangular picture is 12 in. by 16 in. When each dimension is increased by the same amount, the area is increased by 60 in^2. If x represents the number of inches by which each dimension is increased, which equation could be used to find the value for x?



$(16+x)(12+x)=252$

$192+16x+12x+x^2=252$

$192+28x+x^2=252$

$x^2+28x-60=0$

Find zeros!

$\boxed{x=2}$

Pascal's Triangle

Pascal's Triangle is a way to Expand large polynomials without FOILing!.

Draw the triangle:

1 row 1
1 1 row 2
1 2 1 row 3
1 3 3 1 row 4
1 4 6 4 1 row 5
1 5 10 10 5 1 row 6
1 6 15 20 15 6 1 row 7

Step 1: degree + 1 = row and terms used

Step 2: Put in sandwich piece with binomial terms

Step 3: Insert exponents

Step 4: Insert coefficient

Simplif

Example 6: Expand $(a+b)^5$. degree + 1 = row and terms used

$(1)(a)^5 + (5)(a)^4(b)^1 + (10)(a)^3(b)^2 + (10)(a)^2(b)^3 + (5)(a)^1(b)^4 + (1)(b)^5$

$\boxed{= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5}$

Example 7: Expand $(2x+4)^3$.

$)(2x)^3 + (3)(2x)^2(3)^1 + (3)(2x)^1(3)^2 + (1)(4)^3$

$\boxed{= 8x^3 + 36x^2 + 54x + 64}$

You try! Expand $(-3x+2)^4$.

$(-3x)^4 + (4)(-3x)^3(2)^1 + (6)(-3x)^2(2)^2 + (4)(-3x)^1(2)^3 + (1)(2)$

$\boxed{= 81x^4 - 216x^3 + 216x^2 - 96x + 16}$

Day 4: Dividing Polynomials, $f(x) = g(x)$ as a Solution

Polynomial Long Division

Polynomial long division helps us find out if polynomials are factors of other polynomials!

Example 1: Let's review simple long division first.

$$\begin{array}{r} \boxed{32} \\ 21\overline{)672} \\ -63\downarrow \\ \hline 42 \\ -42 \\ \hline 0 \end{array}$$

Now, let's try the same process with polynomials.

$$\begin{array}{r} \boxed{3x+2} \\ 2x+1\overline{)6x^2+7x+2} \\ -6x^2+3x\downarrow \\ \hline 4x+2 \\ -4x-2 \\ \hline 0 \end{array}$$

What was your remainder? 0 This means that 2x + 1 is a factor of $6x^2 + 7x + 2$!

Prove it! Factor $6x^2 + 7x + 2$ like you're used to doing.

12

4, 3

$(6x^2+4x+3x+2)$

$2x(3x+2)+1(3x+2)$

$(2x+1)(3x+2)$ these are the same factors you found with long division

You try!

$$\begin{array}{r} 4x+3 \\ x+5\overline{)4x^2+23x-16} \\ -4x^2+20x\downarrow \\ \hline 3x-16 \\ -3x-15 \\ \hline -31 \end{array}$$

$\boxed{4x+3 \quad R-31}$

You try! This time, be very careful about any missing terms you might have!

Is $x^2 + 1$ a factor of $3x^4 - 4x^3 + 12x^2 + 5$?

$$\begin{array}{r|l} & 3x^2 - 4x + 9 \\ \hline x^2 + 0x + 1 & 3x^4 - 4x^3 + 12x^2 + 0x + 5 \\ & -3x^4 - 0x^3 - 3x^2 \\ \hline & -4x^3 + 9x^2 + 0x \\ & 4x^3 + 0x^2 + 4x \\ \hline & 9x^2 + 4x + 5 \\ & -9x^2 - 0x - 9 \\ \hline & 4x - 4 \end{array}$$

$$\boxed{3x^4 - 4x + 9 \quad R\ 4x - 4}$$

Synthetic Division

Synthetic division can be a great simple tool, but only when The divisor has a leading coefficient of 1.

Example 2: Use synthetic division to divide $x^3 - 14x^2 + 51x - 54$ by $x + 2$.

Step 1: Set $x + 2 = 0$ and solve for x. $x = -2$

Step 2: Put that number in the upper left box and list your coefficients next to it. Bring the first coefficient down.

Step 3: Multiply the coefficient by the box number (the divisor). Add to the next coefficient.

Step 4: Continue multiplying and adding through the last coefficient.

$$\begin{array}{r|rrrr} -2 & 1 & -14 & 51 & -54 \\ & \downarrow & -2 & 32 & -166 \\ \hline & 1 & -16 & 83 & -220 \end{array}$$

$$\boxed{x^2 - 16x + 83 \quad R\ -220}$$

always remainder!

You try! Use synthetic division to determine if $x - 7$ is a factor of $x^3 - 57x + 56$. Watch for missing terms!

$x - 7 = 0$
$x = 7$

$$\begin{array}{r|rrrr} 7 & 1 & 0 & -57 & 56 \\ & \downarrow & 7 & 49 & -56 \\ \hline & 1 & 7 & -8 & 0 \end{array}$$

$$\boxed{x^2 + 7x - 8 \quad R0}$$

yes!

Apply it! The polynomial $x^3 + 7x^2 - 38x - 240$ expresses the volume, in cubic inches, of a shadow box. What are the dimensions of the box? The length is greater than the height.

★ Use graph to find a factor! $x = 6$ $(x-6)$

$$\begin{array}{r|rrrr} 6 & 1 & 7 & -38 & -240 \\ & \downarrow & 6 & 78 & 240 \\ \hline & 1 & 13 & 40 & 0 \end{array}$$

$x^2 + 13x + 40$

8, 5

$(x^2 + 8x)(+5x + 40)$

$x(x+8) + 5(x+8)$

height → $(x+5)(x+8)$ ← length

The Remainder Theorem: If you divide a polynomial by $x - a$, the remainder will be $P(a)$

Example 3: Given that $P = x^5 - 2x^3 - x^2 + 2$, use the Remainder Theorem to find if $x - 3$ is a factor of $P(x)$.

$(3)^5 - 2(3)^3 - (3)^2 + 2$ plug in 3!

$= 182$ $\boxed{x-3 \text{ is NOT a factor!}}$

remainder →

You try! Find the remainder if $P(x) = x^5 - 3x^4 - 28x^3 + 5x + 20$ is divided by $x - 4$.

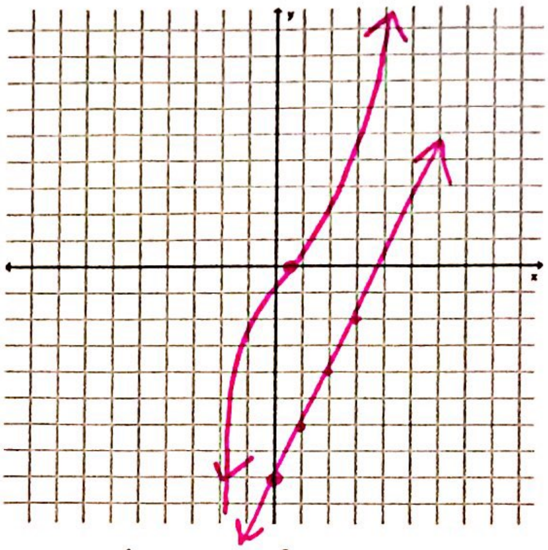
$(4)^5 - 3(4)^4 - 28(4)^3 + 5(4) + 20$ plug in 4!

$1024 - 768 - 1792 + 20 + 20 = \boxed{-1496}$

Finding the Solutions of Two Polynomials, f(x) and g(x)

A few days ago, you worked on graphing polynomials, given their end behavior and multiplicity. But, what if you graphed two at once? What would their solutions be?

Example 4: Sketch $f(x) = 3x^3 + x^2 + 10x - 5$ and $g(x) = 2x - 8$. What is the solution set?



Find where they cross!

$x = \{-.37\}$

You try! What is the solution set for $\frac{1}{4}f(x)$ and $-\frac{3}{4}g(x)$ if $f(x) = 8x^4 + 11x^3 + 5x^2$ and $g(x) = x - 2$? Be careful of your coefficients!

$Y_1: \frac{1}{4}(8x^4 + 11x^3 + 5x^2)$

$Y_2: \frac{-3}{4}(x-2)$

$(-1.4, 2.5)$
$(.6, 1.1)$
$x = \{-1.4, .6\}$

Let's apply! Alphonsus is running through the woods on a path that matches $a(x) = -2(x-5)^4 + 10$. Tyler is running on a path that matches $t(x) = -2x + 8$. When will the two of them see each other? x represents the minutes they have been running.

Intersections: $(3.5, 0.9)$
$(6.7, -5.3)$

They will see each other at 3.5 and 6.7 seconds.