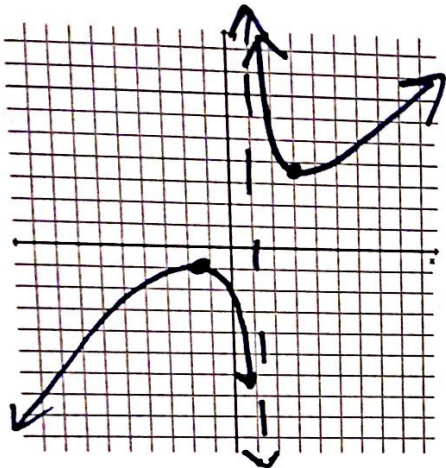


You try! What are all the points of discontinuity?

a) $y = \frac{x^2+4}{x-1}$



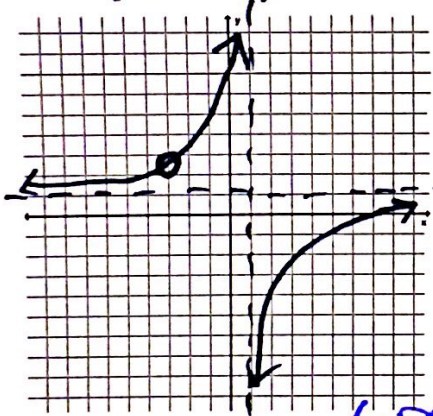
- 1) Find what makes the bottom 0.
- 2) Ask yourself, does it make the top 0 also?
- 3) Yes \rightarrow hole No \rightarrow VA

Domain: Range:

VA: $x=1$ HA: none

no holes

b) $y = \frac{(x-2)(x+3)}{(x+3)(x-1)}$



Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, -2.47] \cup [6.47, \infty)$

$x = -3 \rightarrow$ hole
 $x = 1 \rightarrow$ VA

HA: $y = 1$

Domain: $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$ Range: $(-\infty, 0) \cup (0, 1.25) \cup (1.25, \infty)$

Day 4: Multiplying and Dividing Rational Expressions

There is one goal when it comes to multiplying and dividing rational expressions, and that is to

cancel as many terms as possible!

Example 1: Simplify $\frac{x^2+10x+25}{x^2+9x+20} = \frac{(x+5)(x+5)}{(x+4)(x+5)} = \boxed{\frac{x+5}{x+4}}$

* always factor first!

decide restrictions now!

$x \neq -4, -5$

What are the restrictions on x? (What would make the denominator = 0?) $x \neq -5, -4$

You try! Simplify: a) $\frac{27x^3y}{9x^4y} = \boxed{\frac{3}{x}}$

b) $\frac{-6-3x}{x^2-6x+8} = \boxed{\frac{-3(2+x)}{(x-4)(x-2)}}$

Restrictions: $x \neq 0, y \neq 0$

Restrictions: $x \neq 4, 2$

Domain: _____ Range: _____

Domain: $(-\infty, 2) \cup (2, 4) \cup (4, \infty)$ Range: $(-\infty, 0) \cup (14.85, \infty)$

Multiplying Rational Expressions

Example 2: Multiply $\frac{2x^2+7x+3}{x-4} \cdot \frac{x^2-16}{x^2+8x+15}$

★ factor first!

$$\frac{(2x^2+6x+x+3)}{(2x+1)(x+3)} \cdot \frac{(x-4)(x+4)}{(x+5)(x+3)} = \frac{(2x+1)(x+4)}{x+5}$$

Restrictions: $x \neq 4, -5, -3$

Domain: $(-\infty, -5) \cup (-5, \infty)$ Range: $(-\infty, -19.49] \cup [-2.51, \infty)$

You try! Multiply. a) $\frac{x+7}{7(x+5)} \cdot \frac{x^2-3x-40}{7x+35} \cdot \frac{x-8}{x-8} = \boxed{\frac{x+7}{7}}$

b) $\frac{45x^2}{x-9} \cdot \frac{x^2-5x-36}{3x^3+12x^2} = \boxed{15}$

Restrictions: $x \neq -5, 8$

Restrictions: $x \neq 9, 0, -4$

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Domain: $(-\infty, \infty)$ Range: $[15]$

Dividing Rational Expressions

Example 3: Divide $\frac{4-x}{(3x+2)(x-2)} \div \frac{5(x-4)}{x-2(7y-5)}$

$$\frac{4-x}{(3x+2)(x-2)} \cdot \frac{(x-2)(7y-5)}{5(x-4)} = \frac{(4-x)(7y-5)}{5(3x+2)(x-4)} = \boxed{\frac{-(4-x)(7y-5)}{5(3x+2)(x-4)}}$$

Restrictions: $x \neq \frac{2}{3}, 2, 4$ Domain: _____ Range: _____

$$= \boxed{\frac{-(7y-5)}{5(3x+2)}}$$

What should we always remember when dividing fractions?

keep/change/flip

When should you decide your restrictions?

when everything is factored and after keep/change/flip

$$\begin{array}{ccc} (5x^2 - 10x)(4x + 8) & (25x^2 - 20x)(5x + 4) & (5x^2 - 40x)(x + 8) \\ 5x(x-2) - 4(x-2) & 5x(5x-4) - 1(5x-4) & 5x(x-8) - 1(x-8) \\ (5x-4)(x-2) & (5x-1)(5x-4) & (5x-1)(x-8) \end{array}$$

Restrictions: $x = \frac{-2}{3}, 2, 4$ Domain: _____ Range: _____

You try! Divide a) $\frac{b^2 - 2b - 15}{8b + 20} \div \frac{2}{4b + 10}$

$$\frac{(b+3)(b-5)}{4(2b+5)} \cdot \frac{2(2b+5)}{2} = \frac{(b+3)(b-5)}{4}$$

b) $\frac{10x^2 - 28x + 16}{2x - 4} \div \frac{25x^2 - 25x + 4}{5x^2 - 41x + 8}$

$$\frac{2(5x^2 - 14x + 8)}{2(x-2)} \cdot \frac{25x^2 - 25x + 4}{5x^2 - 41x + 8}$$

$$\frac{2(5x-4)(x-2)}{2(x-2)} \cdot \frac{(5x-1)(x-8)}{(5x-1)(5x-4)} = \boxed{x-8}$$

Restrictions: $b \neq -\frac{5}{2}$
Domain: $(-\infty, \infty)$ Range: $[-4, \infty)$

Restrictions: $x \neq 2, \frac{1}{5}, \frac{4}{5}$
Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Day 5: Adding and Subtracting Rational Expressions

Just like addition and subtraction with all fractions, the main thing to focus on is Canceling
as many terms as possible.

Adding Rational Expressions

Example 1: Add $\frac{3}{x+6} + \frac{7}{x-2}$

Ask yourself how you can make the denominators the same!

$$\frac{3(x-2)}{(x+6)(x-2)} + \frac{7(x+6)}{(x+6)(x-2)} = \frac{3x-6}{(x+6)(x-2)} + \frac{7x+42}{(x+6)(x-2)} = \boxed{\frac{10x+36}{(x+6)(x-2)}}$$

What are the restrictions on x? $x \neq -6, 2$

You try! Add $\frac{7n}{n+1} + \frac{8}{n-7}$

$$\frac{7n}{(n+1)(n-7)} + \frac{8(n+1)}{(n+1)(n-7)} = \frac{7n}{(n+1)(n-7)} + \frac{8n+8}{(n+1)(n-7)}$$

$$= \boxed{\frac{15n+8}{(n+1)(n-7)}}$$

What are the restrictions on x? $x \neq -1, 7$

Example 2: Add $\frac{1}{x^2+5x+4} + \frac{5x}{3x+3}$

Factor first and ask yourself how you can make the denominators the same!

$$\frac{1}{(x+4)(x+1)} + \frac{5x}{3(x+1)} = \frac{1(3)}{3(x+4)(x+1)} + \frac{5x(x+4)}{3(x+4)(x+1)}$$

$$= \frac{3}{3(x+4)(x+1)} + \frac{5x^2+20x}{3(x+4)(x+1)} = \boxed{\frac{5x^2+20x+3}{3(x+4)(x+1)}}$$

What are the restrictions on x? $x \neq -4, -1$

You try! Add $\frac{5n+5}{5n^2+35n-40} + \frac{7}{3n}$

$$\frac{5(n+1)}{5(n^2+7n-8)} + \frac{7}{3n} = \frac{5(n+1)}{5(n+8)(n-1)} + \frac{7}{3n}$$

$$\frac{1}{n+8} + \frac{7}{3n} = \frac{1(3n)}{(n+8)(3n)} + \frac{7(n+8)}{(3n)(n+8)} = \frac{3n}{(n+8)(3n)} + \frac{7n+56}{(3n)(n+8)}$$

$$= \frac{10n+56}{(3n)(n+8)}$$

What are the restrictions on x? $n \neq -8, 1, 0$

Example 3: Subtract $\frac{7}{3} - \frac{8}{12x-8} = \frac{7}{3} - \frac{8}{4(3x-2)} = \frac{7}{3} - \frac{2}{3x-2}$

$$= \frac{7(3x-2)}{3(3x-2)} - \frac{2(3)}{3(3x-2)} = \frac{21x-14}{3(3x-2)} - \frac{6}{3(3x-2)} = \boxed{\frac{21x-20}{3(3x-2)}}$$

What are the restrictions on x? $x \neq \frac{2}{3}$

You try! $\frac{5}{n+5} - \frac{4n}{2n+6}$

$$\frac{5}{n+5} - \frac{2(2n)}{2(n+3)} = \frac{5}{n+5} - \frac{2n}{n+3} = \frac{5(n+3)}{(n+5)(n+3)} - \frac{2n(n+5)}{(n+5)(n+3)} = \frac{5n+15}{(n+5)(n+3)} - \frac{2n^2+10n}{(n+5)(n+3)}$$

$$= \frac{-2n^2-5n+15}{(n+5)(n+3)}$$

What are the restrictions on x? $n \neq -5, -3$

Example 4: Subtract $\frac{5x-4}{2x^2+3x-9} - \frac{x+6}{5x^2+19x+12}$

$$\frac{5x-4}{(2x-3)(x+3)} - \frac{x+6}{(5x+4)(x+3)}$$

$$\frac{5x-4}{(2x-3)(x+3)(5x+4)} - \frac{x+6(2x-3)}{(2x-3)(x+3)(5x+4)} = \frac{5x-4}{(2x-3)(x+3)(5x+4)} - \frac{2x^2+9x-18}{(2x-3)(x+3)(5x+4)}$$

$$= \frac{-2x^2-4x+14}{(2x-3)(x+3)(5x+4)}$$

What are the restrictions on x? $x \neq \frac{3}{2}, -3, -\frac{4}{5}$

You try! Subtract $\frac{7y}{5y^2-125} - \frac{4}{3y+15}$

$$\frac{7y}{5(y+5)(y-5)} - \frac{4}{3(y+5)} = \frac{7y(3)}{5(y+5)(y-5)(3)} - \frac{4(5)(y+5)(y-5)}{5(y+5)(y-5)(3)}$$

$$= \frac{21y}{15(y+5)(y-5)} - \frac{20(y^2-25)}{5(y+5)(y-5)} = \frac{21y-20y^2+500}{5(y+5)(y-5)}$$

What are the restrictions on x? $y \neq -5, 5$

Day 6: Complex Rational Expressions

Simplifying complex rational expressions involves Simplifying the top, then the bottom.

Example 1: Simplify $\frac{\frac{x-2}{3} + \frac{2}{x+1}}{\frac{x-1}{x-1} - \frac{1}{x+1}}$

First simplify the top:

$$\frac{x-2}{x} + \frac{2}{x+1} = \frac{(x-2)(x+1)}{x(x+1)} + \frac{2(x)}{x(x+1)} = \frac{x^2-x-2}{x(x+1)} + \frac{2x}{x(x+1)}$$

$$\boxed{\frac{x^2+x-2}{x(x+1)}}$$

Then, simplify the bottom:

$$\frac{3}{x-1} - \frac{1}{x+1} = \frac{3(x+1)}{(x-1)(x+1)} - \frac{1(x-1)}{(x-1)(x+1)}$$

$$= \frac{3x+3}{(x-1)(x+1)} - \frac{x-1}{(x-1)(x+1)}$$

$$= \boxed{\frac{2x+2}{(x-1)(x+1)}}$$

Then, keep/change/flip:

$$\frac{x^2+x-2}{x(x+1)} \cdot \frac{(x-1)(x+1)}{2x+2} = \frac{(x+2)(x-1)}{x(x+1)} \cdot \frac{(x-1)(x+1)}{2(x+1)} = \boxed{\frac{(x+2)(x-1)^2}{2x(x+1)}}$$

What are the restrictions on x? $x \neq 0, -1$

You try! Simplify $\frac{\frac{16}{m-3} - \frac{4}{m-4}}{\frac{16}{m^2} - \frac{4}{m-3}}$

K/C/F

$$\frac{16m - 52}{(m-3)(m-4)} \cdot \frac{m^2(m-3)}{-m^3+4m^2+16m-48}$$

$$\boxed{\frac{12m^3 - 52m^2}{-m^3+4m^2+16m-48}}$$

TOP

$$\frac{16}{m-3} - \frac{4}{m-4} = \frac{16(m-4)}{(m-3)(m-4)} - \frac{4(m-3)}{(m-3)(m-4)}$$

$$= \frac{16m-64}{(m-3)(m-4)} - \frac{4m-12}{(m-3)(m-4)}$$

$$= \boxed{\frac{12m-52}{(m-3)(m-4)}}$$

BOTTOM

$$\frac{16}{m^2} - \frac{4}{m-3} = \frac{16(m-3)}{m^2(m-3)} - \frac{(m-4)(m^2)}{(m-3)(m^2)}$$

$$= \frac{16m-48}{m^2(m-3)} - \frac{m^3-4m^2}{m^2(m-3)}$$

$$= \boxed{\frac{-m^3+4m^2+16m-48}{m^2(m-3)}}$$

Try one more! Simplify $\frac{\frac{1}{x-1} + \frac{x+3}{x-3} + \frac{1}{x+4}}{\frac{1}{x-1} + \frac{x^2-x}{x-1} + \frac{3x-3}{x-1}}$

TOP

$$\frac{1}{x-1} + \frac{x(x-1)}{x-1} + \frac{3(x-1)}{x-1}$$

$$\frac{1}{x-1} + \frac{x^2-x}{x-1} + \frac{3x-3}{x-1} = \frac{x^2+2x-2}{x-1}$$

BOTTOM

$$\frac{x(x+4)}{x+4} - \frac{3(x+4)}{x+4} + \frac{1}{x+4}$$

$$\frac{x^2+4x}{x+4} - \frac{3x+12}{x+4} + \frac{1}{x+4} = \frac{x^2+x-11}{x+4}$$

$$\boxed{\frac{(x^2+2x-2)(x+4)}{(x-1)(x^2+x-11)}}$$