Warm-up

Complete the two-way table. What is the probability that a student randomly selected from these 5000:

[redacted]	Drive	Bus	Walk	Total
Males	1352	1014	234	2600
Females	1536	816	48	2400
Total	2888	1830	282	5000

1) Is a male or walks to school? $\frac{2600 + 282 - 234}{5000}$ 331/625 52.96%
2) Takes the bus to school and is not a female?
3) Does not walk to school or is not a male?
4) If we know we select a male, what is the probability that male drives to school?

Homework

1a) 100 1b) 1/3 2a) 5/9 2b) 22.22%

3) 7/13 4) 1/6 5) 0.7 6) 2/5

7a) 12812904 7b) 7862400 8) Yes, there are 800

9) 11/221 10) 80 11) 4/7 12) 108

13) 1/6 14a) 1/17 14b) 4/663

14c) 1/221 15) 4/13

Conditional Probability

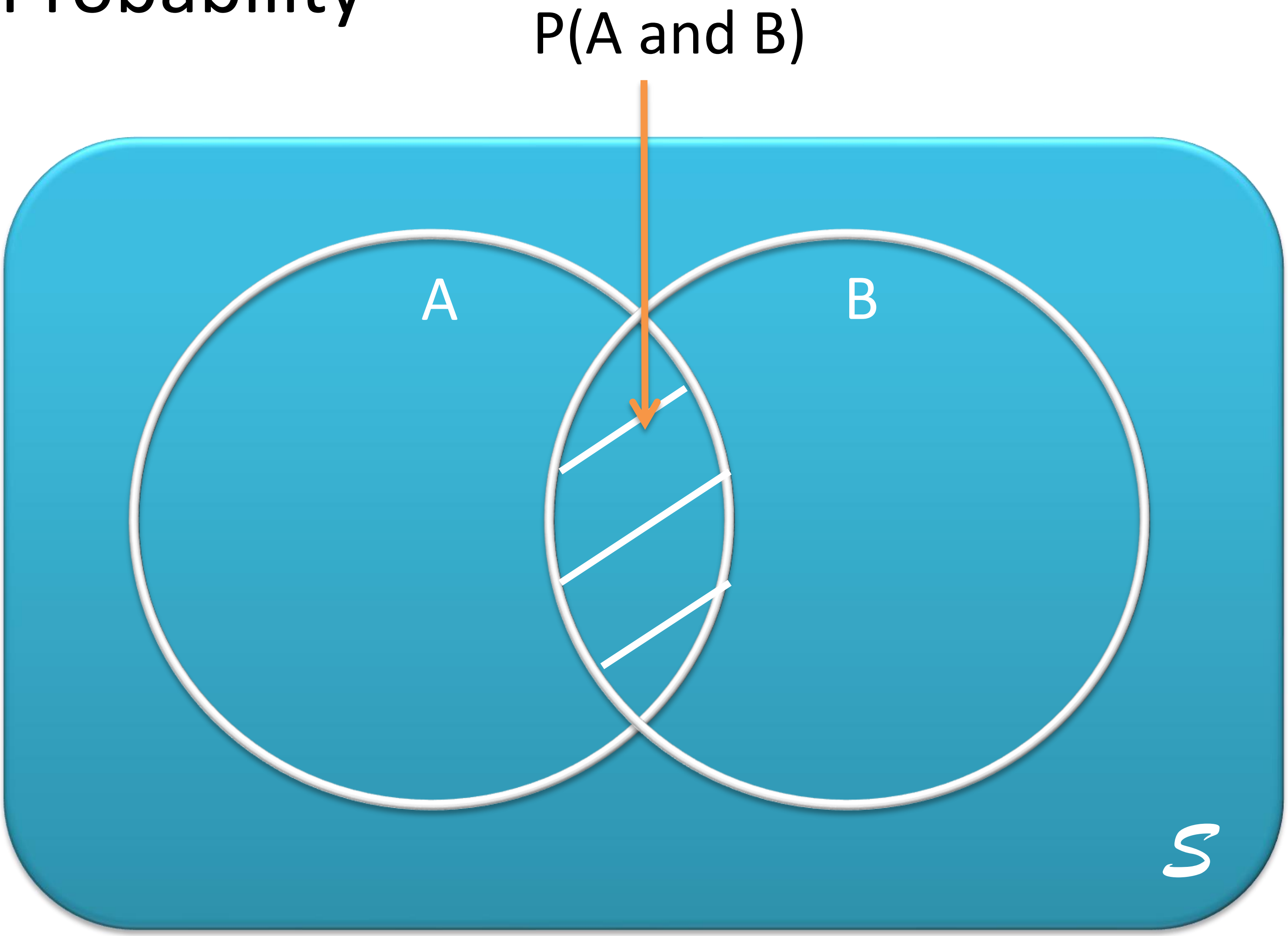
- **Conditional Probability:** A probability where a certain prerequisite condition has already been met.
- For example:
  - What is the probability that a student in the 10th grade is enrolled in biology given that the student is enrolled in CCM2?

Conditional Probability Formula

- The conditional probability of A given B is expressed as P(A | B)

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Joint Probability

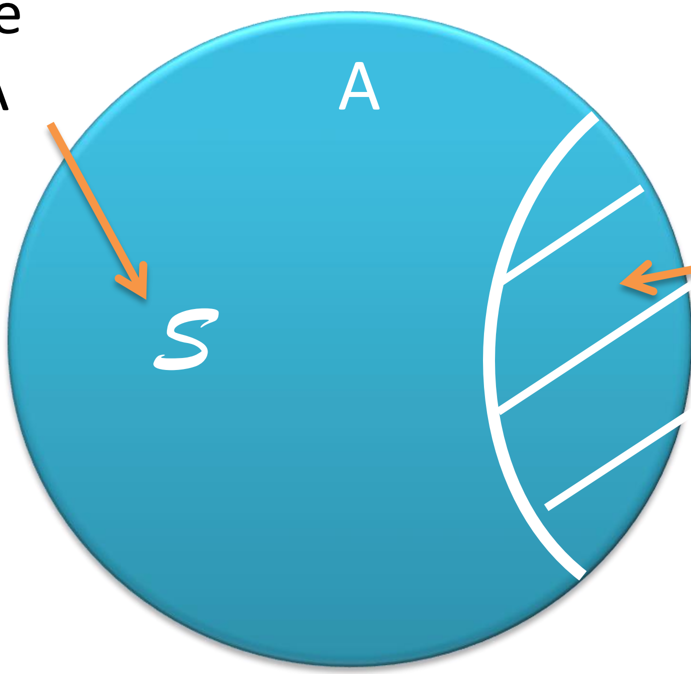


Recall, if there is no overlap the events are *mutually exclusive*.
Another word for mutually exclusive is *disjoint*.

Conditional Probability

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Since Event A has happened, the sample space is reduced to the outcomes in A



P(A and B) represents the outcomes from B that are included in A

Example 1

In New York State, 48% of all teenagers own a skateboard and 39% of all teenagers own a skateboard and roller blades. What is the probability that a teenager owns roller blades given that the teenager owns a skateboard?

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example 2

At a middle school, 18% of all students play football and basketball and 32% of all students play football. What is the probability that a student plays basketball given that the student plays football?

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example 3

In Europe, 88% of all households have a television. 51% of all households have a television and a DVD player. What is the probability that a household has a DVD player given that it has a television?

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example 4

In New England, 84% of the houses have a garage and 65% of the houses have a garage and a back yard. What is the probability that a house has a backyard given that it has a garage?

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Using Two-Way Frequency Tables to Compute Conditional Probabilities

- In CCM1 you learned how to put data in a two-way frequency table (using counts) or a two-way relative frequency table (using percents), and use the tables to find joint and marginal frequencies and conditional probabilities.
- Let's look at some examples to review this.

1. Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the two-way frequency table:

	Bus	Walk	Car	Other	Total
9^{th} or 10^{th}	106	30	70	4	
11^{th} or 12^{th}	41	58	184	7	
Total					

	Bus	Walk	Car	Other	Total
9^{th} or 10^{th}	106	30	70	4	210
11^{th} or 12^{th}	41	58	184	7	290
Total	147	88	254	11	500

Suppose we randomly select one student.

a. What is the probability that the student walked to school?
   - 88/500
   - 17.6%

b. P(9^{th} or 10^{th} grader)
   - 210/500
   - 42%

c. P(rode the bus OR 11^{th} or 12^{th} grader)
   - 147/500 + 290/500 – 41/500
   - 396/500 or 79.2%

	Bus	Walk	Car	Other	Total
9^{th} or 10^{th}	106	30	70	4	210
11^{th} or 12^{th}	41	58	184	7	290
Total	147	88	254	11	500

d. What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?

$P(11^{th} \text{ or } 12^{th} | \text{car})$

* We only want to look at the car column for this probability!

= 11^{th} or 12^{th} graders in cars/total in cars

= 184/254 or 72.4%

The probability that a person is in 11^{th} or 12^{th} grade given that they rode in a car is 72.4%

	Bus	Walk	Car	Other	Total
9^{th} or 10^{th}	106	30	70	4	210
11^{th} or 12^{th}	41	58	184	7	290
Total	147	88	254	11	500

e. What is P(Walk|9th or 10th grade)?

= walkers who are 9^{th} or 10^{th} / all 9^{th} or 10^{th}

= 30/210

= 1/7 or 14.2%

The probability that a person walks to school given he or she is in 9^{th} or 10^{th} grade is 14.2%

4. The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a two-way frequency table that tracks the type of customer and type of ice cream.

	Vanilla	Chocolate	Total
Adult	52		93
Child			
Total		146	224

	Vanilla	Chocolate	Total
Adult	52	41	93
Child	26	105	131
Total	78	146	224

a. Find P(vanilla|adult)
 = 52/93
 = 55.9%

b. Find P(child|chocolate)
 = 105/146
 =71.9%

Examples that are not conditional probability (no formula needed)

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.