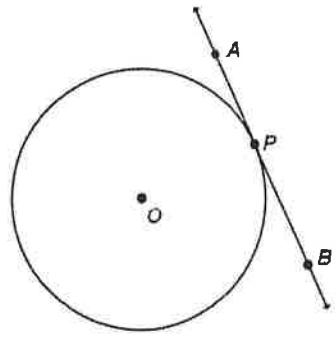
Math 3

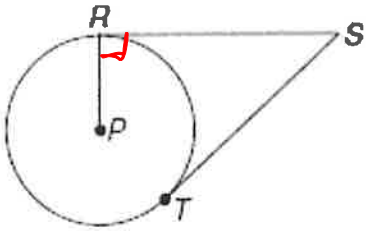
Tangent Lines of Circles

Unit 6

SWBAT solve for unknown variables using theorems about tangent lines of circles.

Tangent to a Circle Ex: (AB)	A line in the plane of the circle that intersects the circle in exactly one point. **Ex:** *Segment AB is a tangent to Circle O.*	
Point of Tangency	The point where a circle and a tangent intersect. **Ex:** *Point P is a point of tangency on Circle O.*	

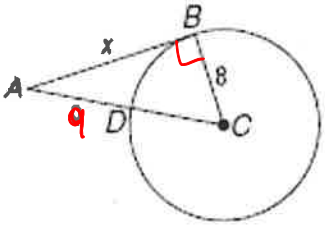
Tangent Theorem 1:	Converse Theorem 1:
If a line is tangent to a circle, then it is perpendicular to the radius draw to the point of tangency.	If a line is perpendicular to the radius of a circle at its endpoint on a circle, then the line is tangent to the circle.



Example: If RS is tangent, then PR $\perp$ RS.

Example 1: Find the measure of x.

a)

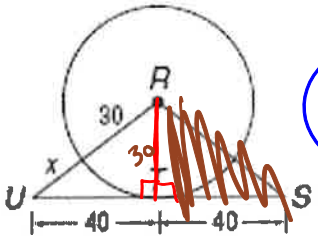


$x^2+8^2=9^2$

$x^2=17$

$x=\sqrt{17}$

b)



$40^2+30^2=(30+x)^2$

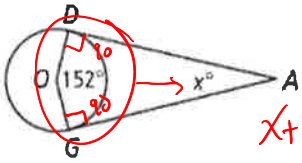
$2500=(x+30)^2$

$50=x+30$

$20=x$

Example 2: Find x. *All segments that appear tangent are tangent to Circle O.*

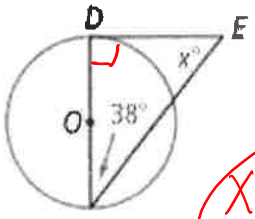
a)



$x+90+90+152=360$

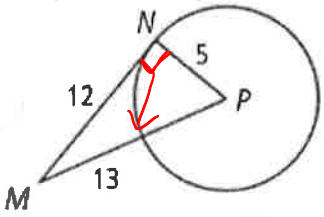
$x=28°$

b)



$x=52°$

Example 3: Is segment MN tangent to Circle O at P? Explain.

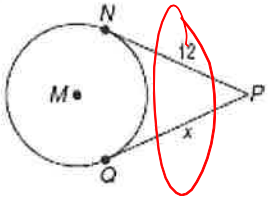


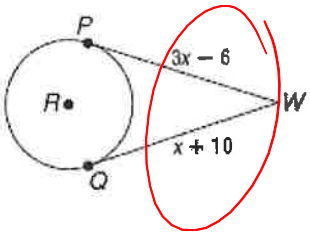
$5^2+12^2=13^2$

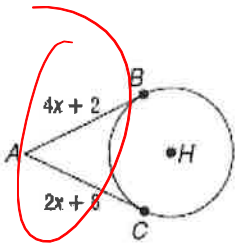
$169=169$ Yes

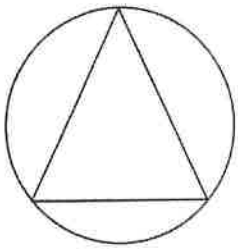
Tangent Theorem 2:	*If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.*

mple 4: Solve for x.

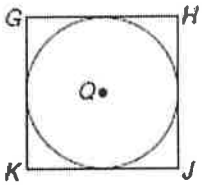


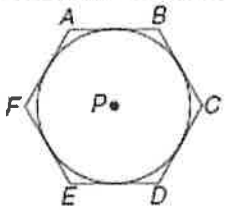




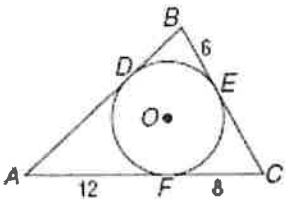
Circumscribed vs. Inscribed		
To ***circumscribe*** is when you draw a figure around another, touching it at points as possible.	To ***inscribe*** is to draw a figure within another so that the inner figure lies entirely within the boundary of the outer.	
Ex: *The circle is circumscribed about the triangle.*	***Ex:*** *The triangle is inscribed in the circle.*	

Tangent Theorem 3: (Circumscribed Polygons)	*When a polygon is circumscribed about a circle, all of the sides of the polygon are tangent to the circle.*



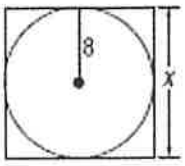


Example 5: Triangle ABC is circumscribed about ⊙O. Find the perimeter of triangle ABC.



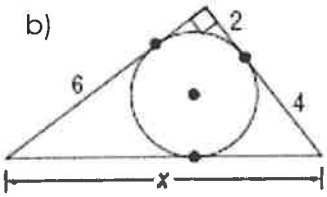
You Try! Find x. Assume that segments that appear to be tangent are tangent.

a)

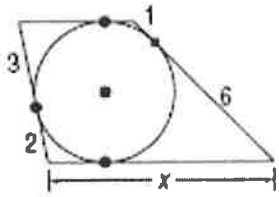


square

b)



c)



Math 3

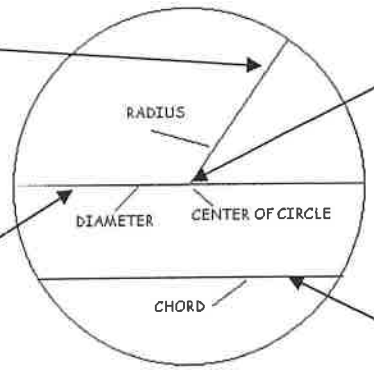
Chords & Arcs of Circles

Unit 6

SWBAT solve for unknown variables using theorems about chords and arcs of circles.

Any segment with endpoints that are the center and a point on the circle is a radius.

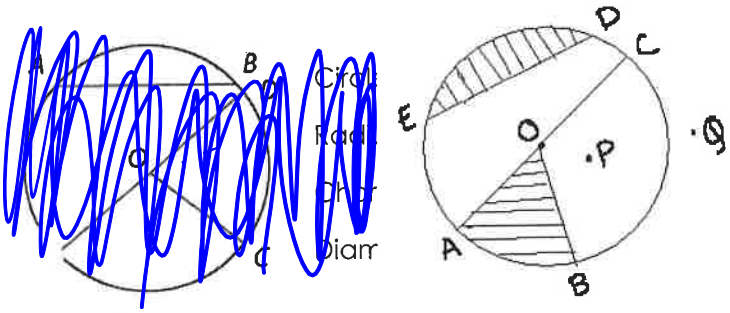
A chord that passes through the center is a diameter of a circle.



The given point is called the center. This point names the circle.

Any segment with endpoints that are on a circle is called a chord.

Example 1: Name the circle, a radius, a chord, and a diameter of the circle.



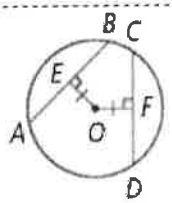
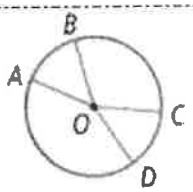
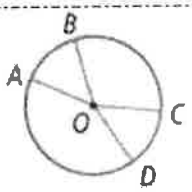
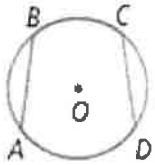
Circle: O

Radius: $\overline{BO}$ $\overline{OC}$ $\overline{AO}$

Chord: $\overline{ED}$

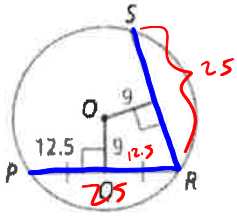
Diameter: $\overline{AC}$

Since a diameter is composed of two radii, then **d = 2r** and **r = d/2**

Theorem 1:	Converse Theorem 1:	
Within a circle or in congruent circles, chords equidistant from the center or centers are congruent. If $OE = OF$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers). If $\overline{AB} \cong \overline{CD}$, then $OE = OF$.	
Theorem 2:	**Converse Theorem 2:**	
Within a circle or in congruent circles, congruent central angles have congruent arcs. If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent arcs have congruent central angles. If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.	
Theorem 3:	**Converse Theorem 3:**	
Within a circle or in congruent circles, congruent central angles have congruent chords. If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent chords have congruent central angles. If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.	
Theorem 4:	**Converse Theorem 4:**	
Within a circle or in congruent circles, congruent chords have congruent arcs. If $\overline{AB} \cong \overline{CD}$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent arcs have congruent chords. If $\overline{AB} \cong \overline{CD}$, then $\overline{AB} \cong \overline{CD}$.	

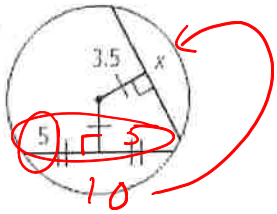
Example 2: The following chords are equidistant from the center of the circle.

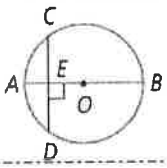
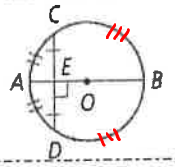
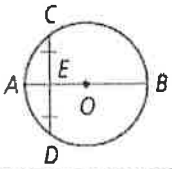
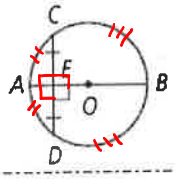
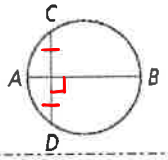
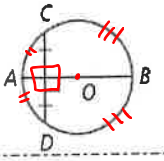
a) What is the length of RS?



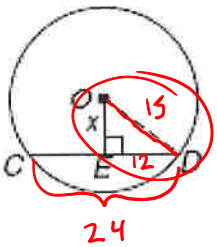
b) Solve for x.

$x = 10$



Theorem 5: In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.	**If . . .** $\overline{AB}$ is a diameter and $\overline{AB} \perp \overline{CD}$ 	**Then . . .** $\overline{CE} \cong \overline{ED}$ and $\widehat{CA} \cong \widehat{AD}$ 
Theorem 6: In a circle, if a diameter bisects a chord that is not a diameter, then it is perpendicular to the chord.	**If . . .** $\overline{AB}$ is a diameter and $\overline{CE} \cong \overline{ED}$ 	**Then . . .** $\overline{AB} \perp \overline{CD}$ 
Theorem 7: In a circle, the perpendicular bisector of a chord contains the center of the circle.	**If . . .** $\overline{AB}$ is the perpendicular bisector of chord $\overline{CD}$ 	**Then . . .** $\overline{AB}$ contains the center of $\odot O$ 

Example 3: In $\odot O$, $\overline{CD} \perp \overline{OE}, OD = 15, and\ CD = 24$. Find x.



$x^2 + 12^2 = 15^2$

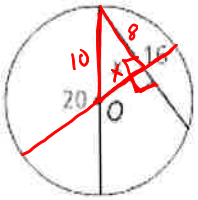
$x = 9$

$x^2 + 144 = 225$

$x^2 = 81$

$x = 9$

Example 4: Find the value of x to the nearest tenth.



$x^2 + 8^2 = 10^2$

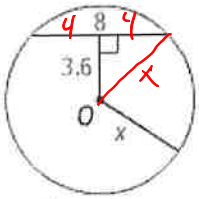
$x^2 + 64 = 100$

$x^2 = 36$

$x = 6$

You Try! Find the value of x to the nearest tenth.

a)



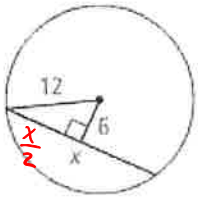
$4^2 + 3.6^2 = x^2$

$5.4 = x$

$12\sqrt{3}$

$x = 20.8$

b)



$\left(\frac{x}{2}\right)^2 + 6^2 = 12^2$

$\frac{x^2}{4} + 36 = 144$

$\frac{x^2}{4} = 108$

$x^2 = 432$

$\sqrt{432}$

$2 \cdot 216$

$2 \cdot 108$

$2 \cdot 54$

$2 \cdot 27$

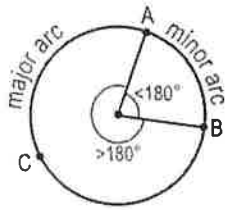
$3 \cdot 9$

$3 \cdot 3$

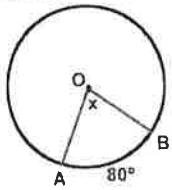
$12\sqrt{3}$

Inscribed Angles

SWBAT apply the rules and theorems of inscribed angles to solve for unknowns.



Major Arc:	Minor Arc:	Semicircle:
An arc of a circle measuring more than or equal to 180°	An arc of a circle measuring less than 180°	An arc of a circle measuring 180°

Central Angle:	A central angle is an angle formed by two intersecting radii such that its vertex is at the center of the circle.	
Central Angle Theorem:	*In a circle, or congruent circles, congruent central angles have congruent arcs.* Central ∠° = arc°	

Example 1: Identify the following in ⊙P at the right. For parts d-f, find the measure of each arc in ⊙P.

a) A semicircle
$\widehat{SQ} = \widehat{STQ} = \widehat{SRQ}$

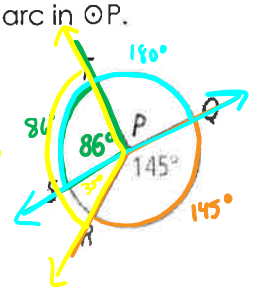
b) A minor arc
$\widehat{ST}$, $\widehat{TQ}$, $\widehat{SR}$, $\widehat{RQ}$, $\widehat{RT}$

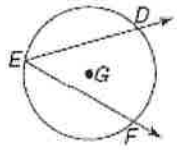
c) A major arc
$\widehat{RTQ}$

d) $\widehat{ST}$
86°

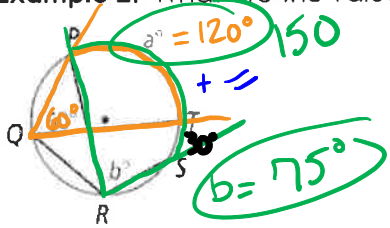
e) $\widehat{STQ}$ 180°

f) $\widehat{RT}$ 121°

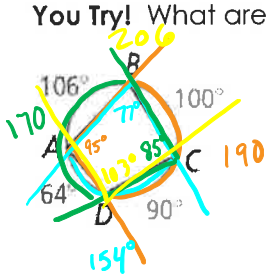


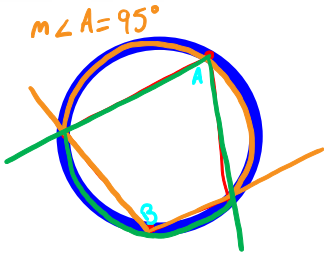
Inscribed Angle:	An inscribed angle is an angle with its vertex "on" the circle, formed by two intersecting chords.	 $m\angle DEF = \frac{1}{2}m\widehat{DF}$
Inscribed Angle Theorem:	*The measure of an inscribed angle is half the measure of its intercepted arc.* angle = 1/2 arc; 2·angle = arc	

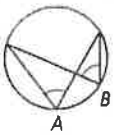
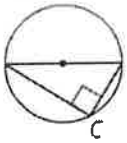
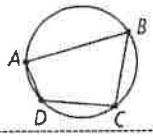
Example 2: What are the values of a and b?



You Try! What are the m∠A, m∠B, m∠C, and m∠D?

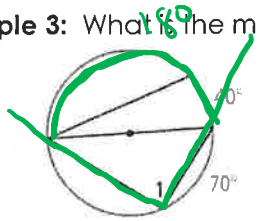




Corollary 1:	Corollary 2:	Corollary 3:
Two inscribed angles that intercept the same arc are congruent. 	An angle inscribed in a semicircle is a right angle. 	The opposite angles of a quadrilateral inscribed in a circle are supplementary. 

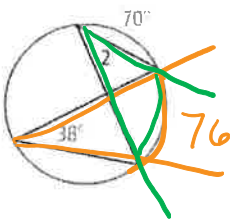
Example 3: What is the measure of each numbered angle?

a)



m∠1 = 90°

b)



m∠2 = 38°

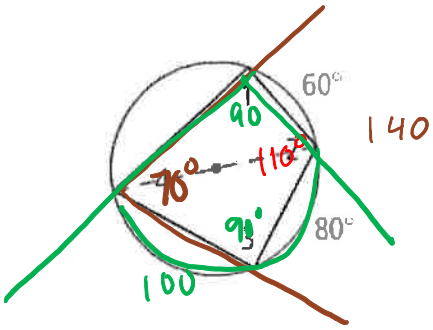
You Try! Find the measure of each numbered angle in the diagram to the right.

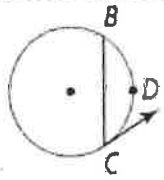
a) $m\measuredangle 1 =$ 90°

b) $m\measuredangle 2 =$ 110°

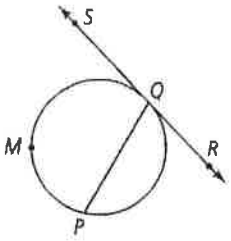
c) $m\measuredangle 3 =$ 90°

d) $m\measuredangle 4 =$ 70°

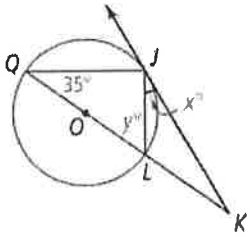


Tangent Chord Angle:	An angle formed by an intersecting tangent and chord has its vertex "on" the circle.	
Tangent Chord Angle Theorem:	*The tangent chord angle is half the measure of the intercepted arc.* **Tangent Chord Angle = ½ (Intercepted Arc)**	$m\angle C = \frac{1}{2} m\widehat{BDC}$

Example 4: In the diagram, $\overleftrightarrow{SR}$ is tangent to the circle at Q. If $m\widehat{PMQ} = 212$, what is the $m\angle PQR$?

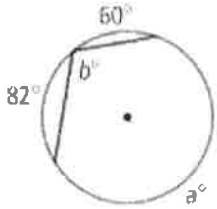


You Try! In the diagram, $\overleftrightarrow{KJ}$ is tangent to ⊙ O. What are the values of x and y?

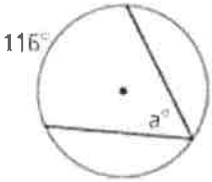


Practice: Find the value of each variable. For each circle, the dot represents the center.

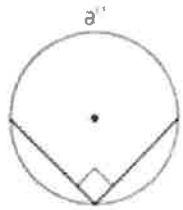
1.



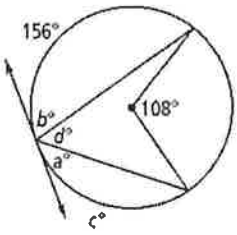
2.



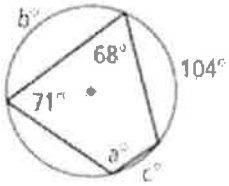
3.



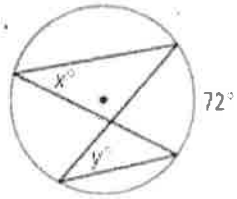
4.



5.



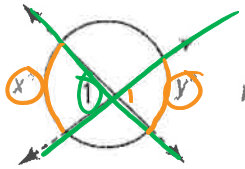
6.



Angle Measures and Segment Lengths

OBJ: Apply the rules and theorems of segments to solve for unknowns.

Theorem 1:	Theorem 2:
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.	The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.

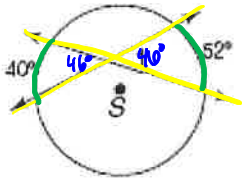


$m\angle 1 = \frac{1}{2}(x + y)$

$m\angle 1 = \frac{1}{2}(x - y)$

Example 1: Find each measure.

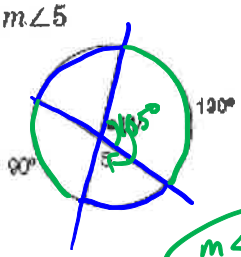
a) $m\angle 1$



52+40= 92÷2

46°

b) $m\angle 5$



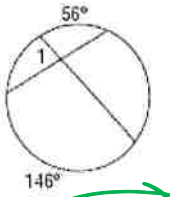
120 + 90

210 ÷ 2

105

m∠5 = 75°

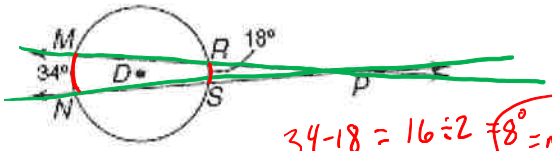
c) $m\angle 1$



m∠1=79°

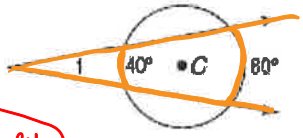
Example 2: Find the following angles.

a) m∠MPN



34-18 = 16÷2 = 8° = m∠MPN

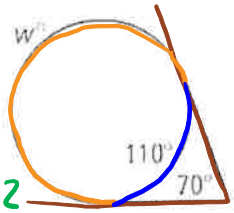
b) $m\angle 1$



80-40=40÷2

m∠1= 20°

c)



250°

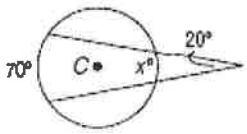
$\frac{(w-110)}{2} = 70$

w-110=140

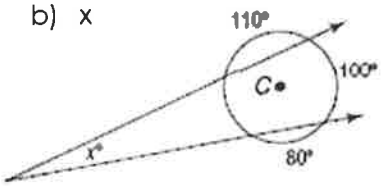
w=250°

You Try! Find the following angles.

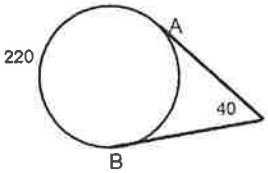
a) x



b) x

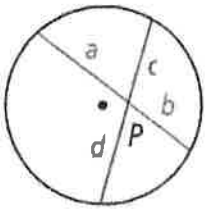


c) Arc AB

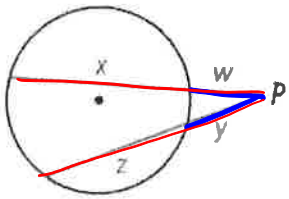


Theorem 3:

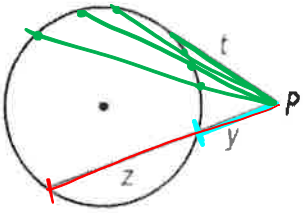
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and the circle.



$a \cdot b = c \cdot d$



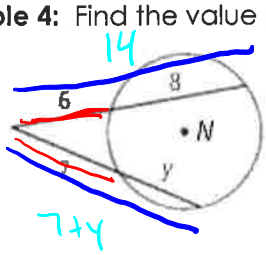
$(w + x)w = (y + z)y$



$(y + z)y = t^2$

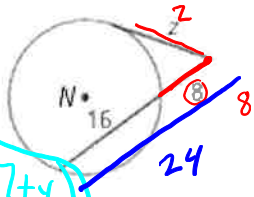
Example 4: Find the value of the variable in ⊙ O.

a)



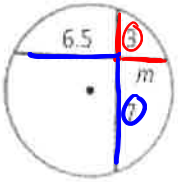
$6 \cdot 14 = 7 \cdot (7+y)$
$84 = 49 + 7y$
$35 = 7y$
$y = 5$

b)



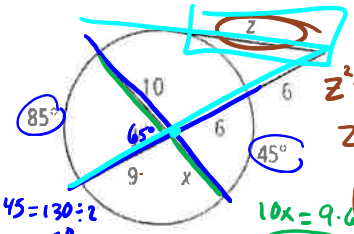
$8 \cdot 24 = z^2$
$192 = z^2$
$96 \quad 2$
$48 \quad 2$
$8 \quad 8$
$3 \quad 2 \quad 4 \quad 2$
$2 \quad 2$
$8\sqrt{3} = z$

c)



$3 \cdot 7 = m \cdot 6.5$
$21 = 6.5m$
$m = \frac{42}{13}$

Try! What is the value of the variable to the nearest tenth?



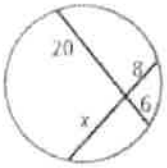
$85 + 45 = 130 \div 2$
$65°$

$z^2 = 6 \cdot 21$
$z^2 = 126$
$z = 11.$
$\sqrt{126}$
$3\sqrt{14}$

$10x = 9 \cdot 6$
$x = 5.4$

Algebra **Find the value of each variable using the given chord, secant, and tangent lengths. If the answer is not a whole number, round to the nearest tenth.** See Problem 3.

15.



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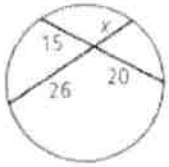

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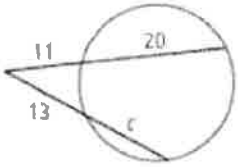

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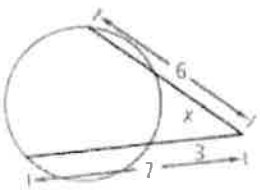

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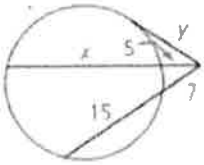

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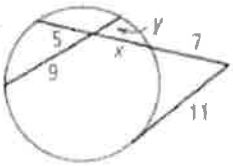


Equations of Circles

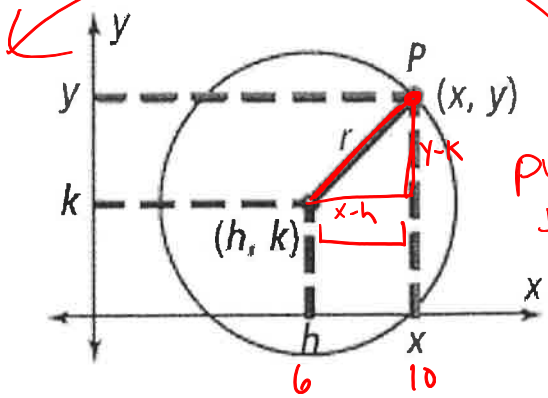
$y = \pm\sqrt{\frac{c-ax^2}{b}}$

SWBAT graph circles on the coordinate plane and write the equations of circles in standard form.

Standard Form of Circles

$ax^2+by^2=c$ $(x-h)^2+(y-k)^2=r^2$

Center:	Radius:	Point on the circle:
(h,k)	r	



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Example 1: Write the equation of a circle with the given information.

a) Center (0,0), Radius=10

~~h = k = r =~~

$x=0$ $y=0$ $r=10$

$x^2+y^2=100$

b) Center (2, 3), Diameter=12

~~h = k = r =~~

$x=2$ $y=3$ $r=6$

$x-2=0$ $y-3=0$ 6^2

$(x-2)^2+(y-3)^2=36$

Example 2: Determine the center and radius of a circle the given equation.

a) $x^2+y^2=\frac{9}{4}$

$x=0$ $y=0$ $\sqrt{r^2}=\sqrt{\frac{9}{4}}$

$(0,0)$ $r=\frac{3}{2}$

b) $(x+3)^2+(y-5)^2=81$

$x+3=0$ $y-5=0$ $r^2=81$

$x=-3$ $y=5$ $r=9$

$(-3,5)$

c) $(x+4)^2+(y+6)^2=1$

$x+4=0$ $y+6=0$ $r^2=1$

$x=-4$ $y=-6$ $r=1$

$(-4,-6)$

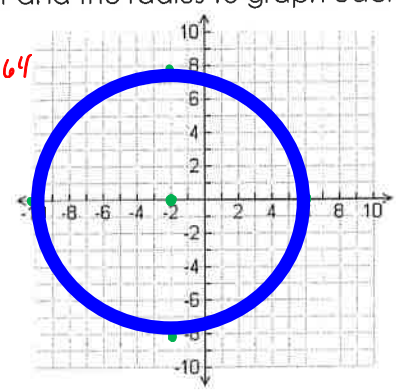
Example 3: Use the center and the radius to graph each circle.

a) $(x+2)^2+y^2=64$

$x+2=0$ $y=0$ $r^2=64$

Center: $(-2,0)$

Radius: $r=8$

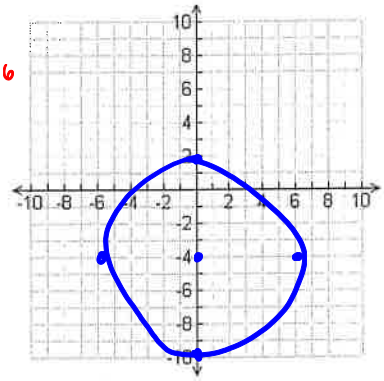


b) $x^2+(y+4)^2=36$

$x=0$ $y+4=0$ $r^2=36$

Center: $(0,-4)$

Radius: $r=6$



Writing an Equation with a Pass-Thru Point
Step 1: Substitute the center (h, k) into the equation
Step 2: Substitute the "pass through point (x, y)" into the equation for x and y.
Step 3: Simplify and solve for r^2.
Step 4: Substitute r^2 back into the equation from Step 1.

Example 4: Write the equation of a circle with a given center (2, 5) that passes through the point (5 ,-1).

$x=2$ $y=5$

$x-2$ $y-5$

$(x-2)^2+(y-5)^2=r^2$

$(5-2)^2+(-1-5)^2=r^2$

$9+36$

$45=r^2$

$(x-2)^2+(y-5)^2=45$

Writing an Equation with Two Points on the Circle	Midpoint Formula
Find the midpoint (radius) between the two endpoints, and then follow steps 1-4.	(average of x, average of y) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example 5: Write the equation of a circle with endpoints of diameter at (-6, 5) and (4, -3).

$-6+4 = -2 \div 2 = -1 = x$ $\quad x+1$

$5-3 = 2 \div 2 = 1 = y$ $\quad y-1$

$(x+1)^2 + (y-1)^2 = 41$

$(x+1)^2 + (y-1)^2 = r^2$

$(-6+1)^2 + (5-1)^2 = r^2$

$25 + 16 = r^2$

$41 = r^2$

Writing the Equation of a Circle in Standard Form	
Step 1:	Group x's and group y's together.
Step 2:	Move any constants to the right side of the equation.
Step 3:	Use complete the square to make a perfect square trinomial for the x's and then again for the y's. **Remember, whatever you do to one side of the equation, you must do to the other!*
Step 4:	Simplify factors into standard form of a circle!

mple 5: Write the equation of a circle in standard form. Then, state the center and the radius.

a) $x^2 + y^2 + 4x - 8y + 16 = 0$

b) $x^2 + y^2 + 6x - 4y = 0$

$(x^2+4x+4) + (y^2-8y+16) = -16 + 4 + 16$

$(x+2)^2 + (y-4)^2 = 4$

$x+2=0$, $x=-2$

$y-4=0$, $y=4$

$r^2=4$

$(-2,4)$

$r=2$

c) $x^2 + y^2 - 6x - 2y + 4 = 0$

d) $x^2 + y^2 + 8x - 10y - 4 = 0$

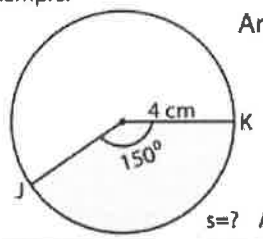
Practice: Arc Length & Area of Sectors

Name: _____

Math 3

$A = \pi r^2$

Example:



Arc length of a sector (s) = $\frac{\theta \times \pi \times r}{180^0}$

$= \frac{150^0 \times 3.14 \times 4}{180^0}$

= **10.47 cm**

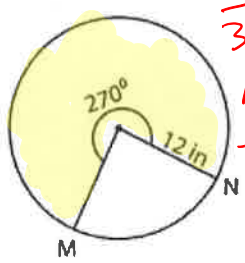
Area = $\frac{\theta \pi r^2}{360}$

$= \frac{150 \cdot 3.14 \cdot 4^2}{360}$

= **20.94 cm²**

Find the length of the arc and area of the shaded region. Round the answer to two decimal places. (π = 3.14)

1)



$\frac{270}{360} = \frac{3}{4}$ portion of circle

$A = \frac{3}{4}\pi(12)^2$

$C = \pi d$

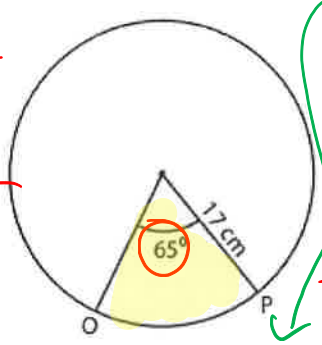
$2\pi r$

$\frac{3}{4} \cdot 2 \cdot \pi \cdot 12$

Length of the arc MN = 56.55 in

Area of a sector = 339.29 in²

2)



$\frac{65}{360}$

$\frac{13}{72}$ portion

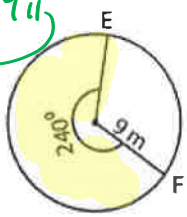
$\frac{13}{72}\pi(17)^2$

$\frac{13}{72} \cdot 34\pi$

Length of the arc OP = 19.29 cm

Area of a sector = 163.93 in²

3)



$\frac{240}{360}$

$\frac{2}{3}$ portion

$\frac{2}{3}\pi(9)^2$

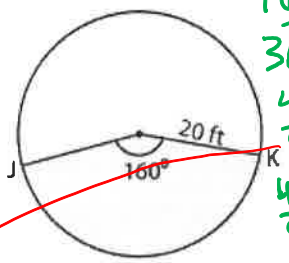
$\frac{2}{3} \cdot 18\pi$

54π

Length of the arc EF = 37.70 m

Area of a sector = 169.65 m²

4)



$\frac{160}{360}$

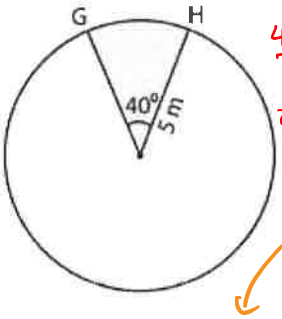
$\frac{4}{9}\pi(20)^2$

$\frac{4}{9} \cdot 2 \cdot \pi \cdot 20$

Length of the arc JK = 55.85 ft

Area of a sector = 558.51 ft²

5)



$\frac{40}{360}$

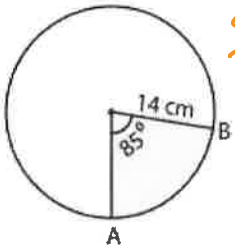
$\frac{1}{9}\pi \cdot 5^2$

$\frac{1}{9} \cdot 2 \cdot \pi \cdot 5$

Length of the arc GH = 3.49 m

Area of a sector = 8.73 m²

6)



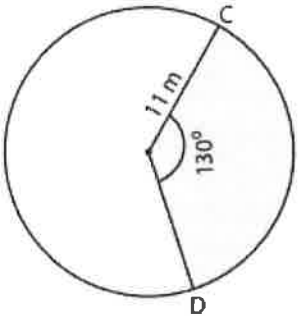
$\frac{85}{360}\pi 14^2$

$\frac{85}{360} \cdot 2\pi \cdot 14$

Length of the arc AB = 20.77 cm

Area of a sector = 145.39 cm²

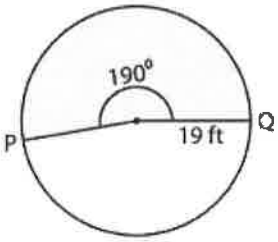
7)



Length of the arc CD = _____

Area of a sector = _____

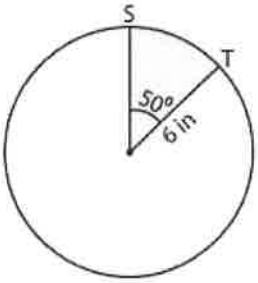
8)



Length of the arc PQ = _____

Area of a sector = _____

9)



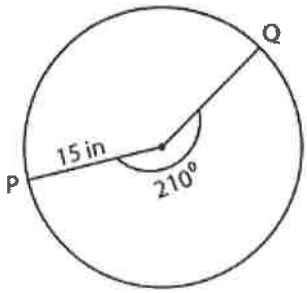
Length of the arc ST = _____

Area of a sector = _____

S = arc length

Find the missing one. Round the radius and central angle to the nearest whole number.
Round the arc length to two decimal places. (use $\pi = 3.14$)

1)

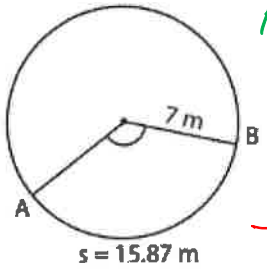


Radius = _____

Central angle = _____

Length of the arc PQ = _____

2)



$\frac{15.87}{14\pi} = \frac{x}{360} \cdot \frac{14\pi}{14\pi}$

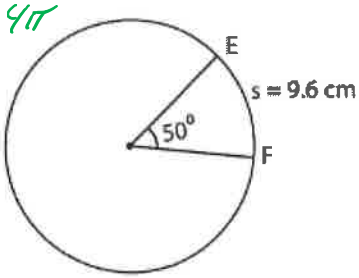
$\cdot 36 = \frac{x}{360}$

Radius = 7m

Central angle = 129.90°

Length of the arc AB = 15.87m

3)



Radius = 11.00 cm

Central angle = 50°

Length of the arc EF = 9.6cm

$9.6 = \frac{50}{360} \cdot 2 \cdot \pi \cdot r$

$r = 11$

Find the missing one. Round the radius and central angle to the nearest whole number.
Round the area to two decimal places. (use $\pi = 3.14$)

1)

$59.66 = \frac{x}{360} \cdot \pi 6^2$

$x =$

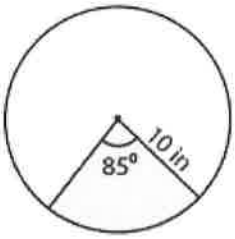
Radius = 6ft

Central angle = 189.9°

Area of a sector = 59.66 ft²

2)

A =

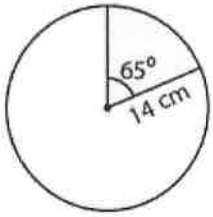


Radius = _____

Central angle = _____

Area of a sector = _____

3)



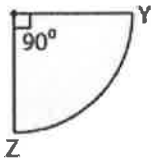
Radius = _____

Central angle = _____

Area of a sector = _____

Find the arc length for each sector. Round the answer to two decimal places. (use π=3.14)

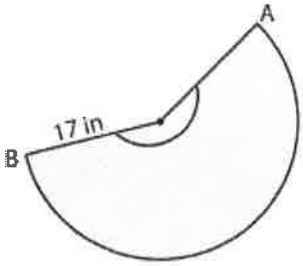
1)



Area = 153.86 cm²

Length of the arc YZ = _____

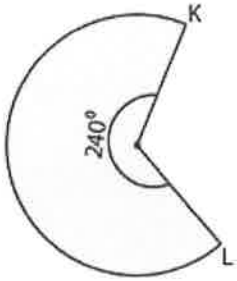
2)



Area = 529.35 in²

Length of the arc AB = _____

3)



Area = 52.33 m²

Length of the arc KL = _____