**Math III Unit 5: INVERSE FUNCTIONS AND LOGARITHMS  
Lauren Winstead, Heritage High School**

**Main topics of instruction:**

1) Inverse Relations and Functions

2) Logarithmic Functions and Properties

3) The number e

4) Compounding Interest

5) Natural Logarithms

**Day 1: Inverse Relations and Functions**

**Example 1:** Find the inverse of .

Step 1: Graph the function.

Step 2: We find by first replacing \_\_\_\_\_\_ with

\_\_\_\_\_\_ and then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Step 3: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Step 4: Graph .

What do you notice about the graphs? They are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of each other across the line \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Is a function? \_\_\_\_\_\_\_\_\_. How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Is a function? \_\_\_\_\_\_\_\_. How do you know? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

|  |  |  |
| --- | --- | --- |
|  | **Domain** | **Range** |
|  |  |  |
|  |  |  |

**You try!**



1. Graph and find the inverse of .

|  |  |  |
| --- | --- | --- |
|  | **Domain** | **Range** |
|  |  |  |
|  |  |  |

1. Graph and find the inverse of .

|  |  |  |
| --- | --- | --- |
|  | **Domain** | **Range** |
|  |  |  |
|  |  |  |

Logarithmic Functions

If , then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**Example 2:** Convert to a log function.

**You try!** a) Convert to a log function. b) Convert to a log function.

**You try!**

1. Evaluate.
2. Evaluate
3. Evaluate
4. Evaluate

**Example 3:** Evaluate .

There are two ways to do this!

|  |  |
| --- | --- |
| **Convert to exponential** | **Change of Base** |
|  |  |

**Example 4:** , what is the inverse of the function? Graph both.



|  |  |  |
| --- | --- | --- |
|  | **Domain** | **Range** |
|  |  |  |
|  |  |  |

**You try!** If , what is the inverse of the function? Graph both.

|  |  |  |
| --- | --- | --- |
|  | **Domain** | **Range** |
|  |  |  |
|  |  |  |

**Example 5:** If , what is ?

|  |  |  |
| --- | --- | --- |
|  | **Domain** | **Range** |
|  |  |  |
|  |  |  |

**You try!** If , what is its inverse?

|  |  |  |
| --- | --- | --- |
|  | **Domain** | **Range** |
|  |  |  |
|  |  |  |

Review of Transformations

**Example 5:** How is translated from ?

Domain:\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Range: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**You try!** How is translated from ?

Domain:\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Range: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Day 2: The Number e and Natural Logarithms**

The number is…

* \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Simplifying *e*

Graphing

Graph .

The inverse of still reflects over \_\_\_\_\_\_\_\_\_\_,

but the function is known as a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

and is written as \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Domain** | **Range** | **Asymptote** |
|  |  |  |  |
|  |  |  |  |

Just like log functions, and can be translated up and down. Fill in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Translations** | **Domain** | **Range** | **Asymptote** |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

**Example 1:** Since and are inverses, what do you think each of the following equals?

**Day 3: Properties of Logarithms**

There are several properties of logarithms that allow us to simplify expressions.

Product Property

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example 1:**

Quotient Property

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example 2:**

Power Property

**Example 3:**

Simplifying Logarithms

**Example 4:** Write the logarithm expression as a single logarithm.

**You try!** Write the logarithm expressions as a single logarithm.

Expanding Logarithms

**Example 5:** Expand the logarithm into multiple logarithms.

**You try!** Expand the logarithm.

**Day 4: Solving Logarithm and Exponential Equations**

The key to solving any logarithmic or exponential equation is to \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**Example 1:** a) Solve . b)

**You try!** a) Solve b) Solve

**Example 2:** Solve

**You try!**

**Example 3:** Solve

**You try!** Solve

**Example 4:** Solve

**You try!**

Solving Systems of Equations with Logarithms

These aren’t too different from solving normal systems!

**Example 1:** **Example 3:**

**You try!**  **You try!**

**Example 2:**

**You try!**

**Day 5: Finite and Continuously Compounded Interest**

Often, people will invest their money at a certain percentage interest and will plan to have that interest delivered to them in pieces throughout the year. It may be compounded once per month, once per day, twice a year, or even constantly.

To find out how much an individual has in his or her account at any given time, we can use one of two formulas.

Compounding a Finite Number of Times

**Example 1:** I have $100 that I invest at a 5% interest rate. My interest is compounded monthly. How much will I have after 3 years?

**You try!** You have $200 that you invest at a 12% interest rate compounded daily. How much will you have after 5 years?

Compounding Continuously

**Example 2:** You invest $500 to be compounded continuously throughout the year at a rate of 25%. How much money will you have after 80 years?

**You try!** Choose your own adventure! Decide how much you will invest, at what rate, and the amount of time you will allow to pass. How much will you make if the interest is compounded continuously?

Investment:

Interest Rate:

Time passed:

**Try one more!**

Suppose you have $4000 to deposit in a bank to earn interest for 10 years, but you have two options. Which option would you choose and why?

*Option 1:* 4.5% annual interest compounded monthly

*Option 2:* 3.75% annual interest compounded continuously

|  |  |  |
| --- | --- | --- |
| Option 1 | Option 2 | Answer |

**Example 3:** Using the same information you used above, what interest rate is required for an investment with continuously compounded interest to triple in 5 years?

**You try!** What interest rate would be required for an investment with continuously compounded interest to quadruple in 6 years?

**Example 4:** Suppose you deposit $3000 in an account that pays 2.5% interest compounded quarterly. How long will it take for the account to reach $6000?

**You try!** You deposit $10,000 into an account that pays 3% interest compounded semiannually. How long will it take for the account to reach $100,000?

**Day 6: Advanced and Piece-wise Functions**

Recall standard form for the four following types of equations:

Linear:

Quadratic:

Exponential:

Polynomial:

Which do you think moves to at the fastest rate? In other words, which is steepest?

Make up your own equations and find out! List the y values for each equation and its corresponding x value.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | Linear | Quadratic | Exponential | Polynomial |
| 0 |  |  |  |  |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

Piece-wise Functions

**Example 1:** Graph the following piece-wise function:

What is

What is

What is



**You try!** Graph the following piece-wise function:

What is

What is

What is

**Example 2:** Graph the following piece-wise function:

What is the distance between the two vertices of the graph?

**You try!** Graph the following piece-wise function:

What is

**Day 7: Geometric Sequences and Series**

Earlier this year, you learned about **arithmetic sequences**, where you had to add a positive or negative number to get to the next number in the sequence.

**Geometric sequences**: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**Common ratio:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**You try!** Which of the following sequences are geometric? What is the common ratio? Find the 8th term of the geometric sequences.

1. 2, 4, 8, 16, … 2) 1, 5, 9, 13, 17,… 3)

You probably figured that out using a **recursive formula**, which relies on \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The equation for a recursive formula looks like: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example 1:** Find the recursive formula for the geometric sequence 4, 12, 36, … where the a1 term is 4 and use it to find the 10th term in the sequence.

But what if I asked you to find the 50th term in a geometric sequence? That would take a lot of work! Instead, you can use an **explicit formula**, which takes you right to the answer.

Explicit formula: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example 2:** The first term of a geometric sequence is 2. The fourth term is -54. Find the second and third terms.

**You try!** Given some terms in a geometric sequence, find the missing terms. Then, write the recursive formula for the sequence.

1. 972, \_\_\_, \_\_\_, \_\_\_, 12 2) 2.5, \_\_\_, \_\_\_, \_\_\_, 202.5 3) 12.5, \_\_\_, \_\_\_, \_\_\_, 5.12

**Example 3:** For the geometric sequence 3, 12, 48, 192,…, find the explicit formula and the indicated term.

1. 5th term 2) 17th term 3) 20th term 4) *n*th term

**You try!** Find the explicit formula and the 10th term of each geometric sequence:

1. a1 = 8, r = 2) a1 =

**Geometric series:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

The sum of a finite geometric series:

**Example 4:** Find the sum of the finite geometric series 3 + 6 + 12 + 24 + …+ 3072.

**You try!** Find the sum of the finite geometric series -5 + -10 + -20 + -40 + …+ -2560

Sums of geometric series can also be expressed in \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**Example 5:** Find the sum of the finite geometric series

**You try!** Find the sum of the finite geometric series

You can also calculate the sum of an infinite geometric series with first term a1 and common ratio . These series \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ to a sum. (If , there is no finite sum for the series. These series \_\_\_\_\_\_\_\_\_\_\_\_\_\_ to an infinite sum.)

**Example 6:** Does the series converge or diverge? If it converges, what is the sum?

What about the series ?

**You try!** Determine if the series converges or diverges, then find the sum if it converges.

1. b)