Transformations of Quadratic Functions (from Vertex Form)

Parent Function: $y = x^2$

Minimum: $a > 0$
maximum: $a < 0$

Vertex: (h, k)

horizontal translations
$x - h$ right
$x + h$ left

$$y = a(x - h)^2 + k$$

vertical translations
$+k$ up
$-k$ down

$0 < |a| < 1$ vertical shrink
$|a| > 1$ vertical stretch
$-a$ reflect over x-axis

Examples: List the vertex, state minimum or maximum, transformations, domain, range, x-intercept, y-intercept, increasing interval, decreasing interval, and end behavior.

1. $y = 2x^2$
2. $y = \frac{1}{3}x^2$
3. $y = -x^2$
4. $y = (x - 3)^2 + 2$
5. $y = (x + 4)^2 - 1$
6. $y = 4(x - 2)^2 - 3$
7. $y = -3(x + 1)^2 - 5$
8. $y = -\frac{2}{3}(x + 3)^2 + 12$
9. $y = -4x^2 + 10$
10. $y = \frac{1}{2}(x - 5)^2$

Quadratic Functions – Parent Function $y = x^2$

Function	Vertex	Min or Max?	Transformations	Domain	Range	x-intercept	y-intercept	Increasing Interval	Decreasing Interval	End Behavior
1.	(0,0)	min	vertical stretch of 2	$(-\infty,\infty)$	$[0,\infty)$	(0,0)	(0,0)	$(0,\infty)$	$(-\infty,0)$	As $x\to\infty, y\to\infty$. As $x\to-\infty, y\to\infty$.
2.										
3.										
4.										
5.										
6.										
7.										
8.	(-3,12)	max	reflect over x-axis, vertical shrink of $\frac{2}{3}$, left 3, up 12.	$(-\infty,\infty)$	$(-\infty,12]$	$\approx$ (-7.24,0) (1.24,0)	(0,6)	$(-\infty,-3)$	$(-3,\infty)$	As $x\to-\infty, y\to-\infty$. As $x\to\infty, y\to-\infty$.
9.	(0,10)	max	reflect over x-axis, vertical stretch of 4, up 10	$(-\infty,\infty)$	$(-\infty,10]$	$\approx$ (-1.58,0) (1.58,0)	(0,10)	$(-\infty,0)$	$(0,\infty)$	As $x\to-\infty, y\to-\infty$. As $x\to\infty, y\to-\infty$.
10.	(5,0)	min	vertical shrink of $\frac{1}{2}$, right 5	$(-\infty,\infty)$	$[0,\infty)$	(5,0)	(0,12.5)	$(5,\infty)$	$(-\infty,5)$	As $x\to-\infty, y\to\infty$. As $x\to\infty, y\to\infty$.

Transformations of Square Root Functions

Parent Function: $y = \sqrt{x}$

starting point: (h, k)

horizontal translations
$x-h$ right
$x+h$ left

$$y = a\sqrt{x - h} + k$$

vertical translations
$+k$ up
$-k$ down

$0<|a|<1$ vertical shrink
$|a|>1$ vertical stretch
$-a$ reflect over x-axis

Examples: List the starting point, transformations, domain, range, x-intercept, y-intercept, increasing interval, decreasing interval, and end behavior.

1. $y = 2\sqrt{x}$
2. $y = \frac{1}{3}\sqrt{x}$
3. $y = -\sqrt{x}$
4. $y = \sqrt{x-3} + 2$
5. $y = \sqrt{x+4} - 1$
6. $y = 4\sqrt{x-2} - 3$
7. $y = -3\sqrt{x+1} - 5$
8. $y = -\frac{2}{3}\sqrt{x+3} + 12$
9. $y = -4\sqrt{x} + 10$
10. $y = \frac{1}{2}\sqrt{x-5}$

Notice that these functions do not have a vertex!

Square Root Functions – Parent Function $y = \sqrt{x}$

Function	First Point	Min or Max?	Transformations	Domain	Range	x-intercept	y-intercept	Increasing Interval	Decreasing Interval	End Behavior
1.	(0,0)	min	vertical stretch of 2	$[0,\infty)$	$[0,\infty)$	(0,0)	(0,0)	$(0,\infty)$	never	As $x \to 0$, $y \to 0$; As $x \to \infty$, $y \to \infty$.
2.	(0,0)	min	vertical shrink of $\frac{1}{3}$	$[0,\infty)$	$[0,\infty)$	(0,0)	(0,0)	$(0,\infty)$	never	As $x \to 0$, $y \to 0$; As $x \to \infty$, $y \to \infty$.
3.										
4.										
5.										
6.										
7.										
8.										
9.	(0,10)	max	reflect over x-axis, vertical stretch of 4, up 10	$[0,\infty)$	$(-\infty,0]$	(6.25, 0)	(0,10)	never	$(0,\infty)$	As $x \to \infty$, $y \to -\infty$; As $x \to 0$, $y \to 10$
10.	(5,0)	min	vertical shrink of $\frac{1}{2}$, right 5	$[5,\infty)$	$[0,\infty)$	(5,0)	none	$(5,\infty)$	never	As $x \to \infty$, $y \to \infty$; As $x \to 5$, $y \to 0$

Transformations of Inverse Variation Functions

Parent Function: $y = \frac{1}{x}$

intersection of asymptotes: (h, k)

Vertical asymptote: $x = h$

horizontal asymptote: $y = k$

$0 < |a| < 1$ vertical shrink (remember fractions use numerator + denominator ↓ $y = \frac{2}{3x}$)

$|a| > 1$ vertical stretch

$-a$ reflect over x-axis

$$y = \frac{a}{x - h} + k$$

$+k$ vertical translations

$+K$ up

$-K$ down

horizontal translations

$x - h$ right

$x + h$ left

Examples: List the transformations, domain, range, horizontal asymptote, vertical asymptote, increasing interval, decreasing interval, and end behavior.

1. $y = \frac{2}{x}$

2. $y = \frac{1}{3x}$

3. $y = -\frac{1}{x}$

4. $y = \frac{1}{x-3} + 2$

5. $y = \frac{1}{x+4} - 1$

6. $y = \frac{4}{x-2} - 3$

7. $y = -\frac{3}{x+1} - 5$

8. $y = -\frac{2}{3(x+3)} + 12$

9. $y = -\frac{4}{x} + 10$

10. $y = \frac{1}{2(x-5)}$

Inverse Variation Functions – Parent Function $y = \frac{1}{x}$

Function	Min or Max?	Transformations	Domain	Range	Horizontal Asymptote	Vertical Asymptote	Increasing Interval	Decreasing Interval	End Behavior
1.	none	vertical stretch of 2	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0)\cup(0,\infty)$	$y=0$	$x=0$	never	$(-\infty,0)\cup(0,\infty)$	As $x\to-\infty, y\to0$. As $x\to\infty, y\to0$.
2.	none	vertical shrink of $\frac{1}{3}$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0)\cup(0,\infty)$	$y=0$	$x=0$	never	$(-\infty,0)\cup(0,\infty)$	As $x\to-\infty, y\to0$. As $x\to\infty, y\to0$.
3.									
4.									
5.									
6.									
7.									
8.									
9.	none	reflect over x-axis, vertical stretch of 4, up 10	$(-\infty,0)\cup(0,\infty)$	$(-\infty,10)\cup(10,\infty)$	$y=10$	$x=0$	$(-\infty,0)\cup(0,\infty)$	never	As $x\to-\infty, y\to10$. As $x\to\infty, y\to10$.
10.	none	vertical shrink of $\frac{1}{2}$, right 5	$(-\infty,5)\cup(5,\infty)$	$(-\infty,0)\cup(0,\infty)$	$y=0$	$x=5$	never	$(-\infty,5)\cup(5,\infty)$	As $x\to-\infty, y\to0$. As $x\to\infty, y\to0$.