Unit 4: Radical and Reciprocal Equations & Functions

By the end of the unit students will be able to:

- Extend the properties of exponents to rational exponents.
- Solve equations using radicals and reciprocal functions
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Day	Date	Lesson	Assignment	Check
1	Friday Oct. 6th	Monomial Rules / Properties of Exponents	Homework 4-1	
2	Monday Oct. 9th	Types of Variation	Homework 4-2	
3	Tuesday Oct. 10th	Systems involving Inverse Variation	Homework 4-3	
4	Wednesday Oct. 11th	Solving Radical Equations	Homework 4-4	
5	Thursday Oct. 12th	Solving Radical Equations	TBD	
6	Friday Oct. 13th	**Review and Quiz**		
7	Monday Oct. 16th	Graphing Radical Functions	Homework 4-7	
8	Tuesday Oct. 17th	Graphing Reciprocal Functions	Homework 4-8	
9	Wednesday Oct. 18th	Review for Test	Review Sheet	
10	Thursday Oct. 19th	**Unit 4 Test**	Unit 5 Placemat	

Homework 4-1

More Properties of Exponents

Date_____ Period__

Simplify. Your answer should contain only positive exponents.

1) $\left(x^{-2}x^{-3}\right)^4$ $\frac{1}{x^{20}}$

2) $\left(x^4\right)^{-3} \times 2x^4$ $\frac{2x^4}{x^{12}}$ $\frac{2}{x^8}$

3) $\left(n^3\right)^3 \times 2n^{-1}$ $2n^8$

4) $(2v)^2 \times 2v^2$ $8v^4$

5) $\frac{2x^2y^4 \times 4x^2y^4 \times 3x}{3x^{-3}y^2}$ $8x^8y^6$

6) $\frac{2y^3 \times 3xy^3}{3x^2y^4}$ $\frac{2y^2}{x}$

7) $\frac{x^3y^3 \times x^3}{4x^2}$ $\frac{x^4y^3}{4}$

8) $\frac{3x^2y^2}{2x^{-1} \times 4yx^2}$ $\frac{3xy}{8}$

9) $\frac{x}{\left(2x^0\right)^2}$ $\frac{x}{4}$

10) $\frac{2m^{-4}}{\left(2m^{-4}\right)^3}$ $\frac{1m^8}{4}$

Variation Practice

For each table, determine whether *y* varies directly with *x*, indirectly with *x*, or neither. Write an equation to model the situation.

1.

x	y
2	14
3	21
5	35

2.

x	y
27	9
30	10
60	20

3.

x	y
3	9
4	10
5	11

4.

x	y
.1	3
3	.1
6	.005
24	.00125

Determine whether *y* varies directly with *x*. If so, find the constant of variation and find y when x = 4.

5. y = 12x

6. y – 6x = 0

7. Y = 4x – 3

8. If j varies jointly as g and the cube of h, and j = 200 when g = 5 and h = 4, find j when g = 3 and h = 6.

9. The average number of phone calls per day between two cities has found to be **jointly proportional** to the populations of the cities, and **inversely proportional** to the square of the distance between the two cities. The population of Charlotte is about 1,500,000 and the population of Nashville is about 1,200,000, and the distance between the two cities is about 400 miles. The average number of calls between the cities is about 200,000.

(a) **Find the k and write the equation of variation.**

(b) The average number of daily phone calls between Charlotte and Indianapolis (which has a population of about 1,700,000) is about 134,000. **Find the distance between the two cities.**

Suppose that x and y vary inversely. Write a function that models each inverse variation. Graph the function and find y when x = 10.

10. x =1 when y = 11

11. x = -4/15 when y = -105

12. X = 1 when y = 1

11. Heart rates and life spans of most mammals are inversely related. A cat lives for about 15.2 years on average and has a heart rate of 126 beats per minute.

a. What is the constant of variation?

b. A hamster has a heart rate of about 634 beats per minute. About how long will a hamster live?

c. An elephant lives for about 70 years. About how many times per minute does an elephant's heart beat?

12. Two gears are used to operate a machine. Gear A has 60 teeth and Gear B has 45 teeth. The speed at which you turn Gear A is 5400 rpm. The number of teeth and speed in rpm are inversely related.

a. What is the constant of variation?

b. At what speed will Gear B turn?

13. The grade you earn in math varies inversely with the number of minutes per night you watch television. If you watch 90 minutes per night, you get a 60 in math.

a. What is the constant of variation?

b. How much television can you watch if you want to make a 70?

c. You cut back on your television to only 75 minutes a night, what grade will you make in math?

14. The amount of water that has leaked from a faucet varies directly with time. In 2 hours, 10 gallons of water leak.

a. Describe what happens to the amount of water as time increases.

b. What is the constant of variation?

c. How much water leaks in 100 hours?

d. How long does it take for 100 gallons to leak?

Honors Math 2 – Fall 2017

HW 4-2 Direct, Inverse and Joint Variation

State whether each equation represents a direct, inverse, or joint variation. Name the constant of variation.

1. $y = 2x$ — Direct, $K=2$

2. $\frac{x}{5} = y$ — Direct $K=\frac{1}{5}$

3. $xy = 12$ — $y=12/x$, Inverse, $K=12$

4. $D = \frac{3}{4}gh$ — joint, $K=\frac{3}{4}$

Write an equation for each statement. Then solve the equation.

5. If y varies directly as x and y = 15 when x = 3, find y when x = 12.

$y=kx$ $\quad y=5x$

$15=k\cdot 3$ $\quad y=5\cdot 12$

$k=5$ $\quad y=60$

6. If y varies inversely as x and y = 2 when x = 8, find x when y = 14.

$y=\frac{k}{x}$ $\quad y=\frac{16}{x}$

$2=\frac{k}{8}$ $\quad 14=\frac{16}{x}$

$16=k$ $\quad 14x=16$

$x=16/14=8/7$

7. Suppose y varies jointly with x and z. If y = 20 when x = 2 and z = 5, find y when x = 14 and z = 8.

$y=kxz$ $\quad y=2xz$

$20=k\cdot 2\cdot 5$ $\quad y=2\cdot 14\cdot 8$

$20=10k$ $\quad y=224$

$2=k$

8. If y varies inversely as x and x = 7 when y = 21, find y when x = 42.

$y=\frac{k}{x}$ $\quad y=147/x$

$21=k/7$ $\quad y=147/42$

$k=147$ $\quad y=7/2$

9. The frequency of a vibrating string varies inversely as its length. A string 3 feet long vibrates 175 cycles per second. Find the frequency of a 5 foot string.

$f=k/\ell \Rightarrow 175=k/3 \Rightarrow k=525$

$f=525/5=105$ cycles/sec

10. The force of the wind blowing on a vertical surface varies jointly as the area of the surface and the square of the velocity. If a wind blowing at 50 mph exerts a force of 75 pounds on a surface of 500 ft^2, how much force will a wind of 75 mph place on a surface of 10 ft^2?

$f=kav^2$

$75=k(500)(50)^2 \Rightarrow k=.00006$

$F=.00006\cdot 10\cdot 75^2=3.375$ lb

11. The time required to process a shipment of goods at Wal-Mart varies directly with the number of items in the shipment and inversely with the number of workers assigned. If 15,000 items can be processed by 8 workers in 10 hours, then how long would it take 12 workers to process 20,000 items?

$T=\frac{ka}{w} \Rightarrow 10=\frac{k\cdot 15000}{8}$

$k=8/1875$

$T=\frac{8\cdot 20000}{1875\cdot 12}=7.1$ hours

Homework 4-4

1.) $4x^{3/2} - 5 = 103$

$4\sqrt{x^3} - 5 = 103$

$4\sqrt{x^3} = 108$

$\sqrt{x^3}^2 = 27^2$

$\sqrt[3]{x^3}^3 = \sqrt[3]{27^2}$

$x = 9$

2.) $\sqrt{x} + 6 = x$

$-6 \quad -6$

$\sqrt{x}^2 = (x-6)^2$

$x = x^2 - 12x + 36$

$0 = x^2 - 13x + 36$

$(x-9)(x-4)$

$x = 9 \quad x = 4$

3.) $(7x-3)^{1/2} = 5$

4.) $\sqrt{x-3} - \sqrt{x} = 3$

$\sqrt{x-3}^2 = (\sqrt{x}+3)^2$

$x - 3 = x + 6\sqrt{x} + 9$

$-x \quad -x$

5.) $5\sqrt{x} + 2 = 12$

6.) $\sqrt[3]{2x-4} = -2$

$-3 = 6\sqrt{x} + 9$

$-12 = 6\sqrt{x}$

$-2 = \sqrt{x}$

$4 = x$ No Soln

7.) $3(2x+4)^{4/3} = 48$

$\frac{3\sqrt[3]{(2x+4)^4}}{3} = \frac{48}{3}$

$\sqrt[3]{(2x+4)^4}^3 = 16^3$

$\sqrt[4]{(2x+4)^4} = \sqrt[4]{16^3}$

$2x + 4 = \pm 8$

$-4 \quad -4$

$\frac{2x}{2} = \frac{\pm 8 - 4}{2}$

$\frac{8-4}{2} = 2$

$\frac{-8-4}{2} = -6$

8.) $\sqrt{12x+13} = 2x+1$

9.) $\sqrt{7x-6} = \sqrt{5x+2}$

$\sqrt{7x-6}^2 = \sqrt{5x+2}^2$

$7x - 6 = 5x + 2$

$2x = 8$

$x = 4$

10.) $(x-2)^{2/3} - 4 = 5$

$\sqrt[3]{(x-2)^2}^3 = 9^3$

$\sqrt{(x-2)^2} = \sqrt{9^3}$

$x - 2 = \pm\sqrt{9^3}$

$+2 \quad +2$

$x = 2 \pm \sqrt{9^3}$

2 ± 27

$29 \quad -25$

11.) The velocity of a free-falling object is given by $V = \sqrt{2gh}$ where **h** is the distance in feet the object has fallen and **g** is acceleration due to gravity in feet per second squared. The value of **g** depends on your altitude. If an object hits the ground with a velocity of 25 feet per second, from what height was it dropped in each of the following situations?

a.) You are standing on earth, so g = 32 ft/s^2.

b.) You are on a space shuttle, so g = 29 ft/s^2.

c.) You are on the moon, so g = 0.009ft/s^2.

$25 = \sqrt{64h}$

$\frac{25}{8} = \frac{8\sqrt{h}}{8}$

$\frac{25^2}{8} = \sqrt{h}^2$

$\frac{625}{64} = h$

9.77 ft

HW 4 – 5 Radical Functions

For the following functions ***describe the transformation,*** *state the* ***End Behavior*** *and find the* ***domain and range****.*

1. $y = \sqrt{x+5}$
T:
D:
R:
EB:
Increase Interval
Decrease Interval

2. $y = \sqrt[3]{x} - 3$
T:
D:
R:
EB:
Increase Interval
Decrease Interval

3. $y = 4\sqrt{x+5}$
T:
D:
R:
EB:
Increase Interval
Decrease Interval

4. $y = -2\sqrt[3]{x+1} - 4$
T:
D:
R:
EB:
Increase Interval
Decrease Interval

5. $y = \frac{1}{5}\sqrt{x+4}$
T:
D:
R:
EB:
Increase Interval
Decrease Interval

6. $y = -5\sqrt[3]{x+5} - 7$
T:
D:
R:
EB:
Increase Interval
Decrease Interval

7. $y = -\frac{1}{4}\sqrt{x-6} + 2$
T:

D:
R:

EB:

Increase Interval
Decrease Interval

8. $y = \sqrt[3]{x+4} + 6$
T:

D:
R:

EB:

Increase Interval
Decrease Interval

9. $y = \frac{2}{5}\sqrt{x-1} + 3$
T:

D:
R:

EB:

Increase Interval
Decrease Interval

****On your own paper practice graphing at least 3 of these ☺**

Homework 4-8 Reciprocal Functions (inverse variation)

Write an equation for a translation of $y = -\frac{3}{x}$ that has the given asymptotes.

1. x = 2, y = 1
2. x = -3, y = 0
3. x = 4, y = -2
4. x = 0, and y = 6

For each of the following functions: a) **graph each equation on your own graph paper**, b) list the horizontal and vertical asymptotes, c) State the domain and range of the function, d) state the end behavior, e) state the intervals of increasing and decreasing.

5. $y = \frac{3}{x-1} + 2$
6. $y = \frac{1}{x-2} + 4$
7. $y = \frac{2}{x-5}$
8. $y = \frac{5}{x} + 1$

Solve the following:

1. $(4^x)^3 = 4096$
2. $(3^{2x})^2 = 81$
3. $(5^x)^3 = 15625$
4. $\frac{3^3}{3^x} = 243$
5. $\frac{2^{2x}}{2^3} = 32$
6. Write $4^5 = 1024$ in logarithmic form

   Answer: _____

7. Solve for *x*:

   $\log_5 x = 2$

   Answer: _____

8. Calculate:

   $\log_3 234$

   Answer: _____

9. Calculate:

   $\frac{\log 100}{\log 10}$

   Answer: _____

10. Calculate:

    $\frac{\log_3 9}{\log_2 8}$

    Answer: _____