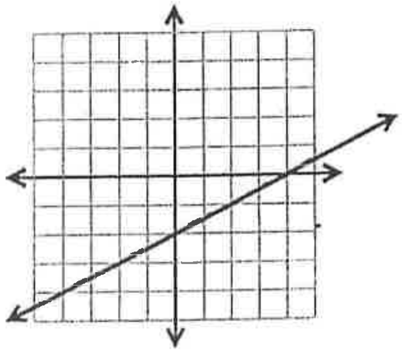
Day 1: Graphing Equations of Lines & Function Notation

SLOPE INTERCEPT FORM OF A LINE: $y = mx + b$

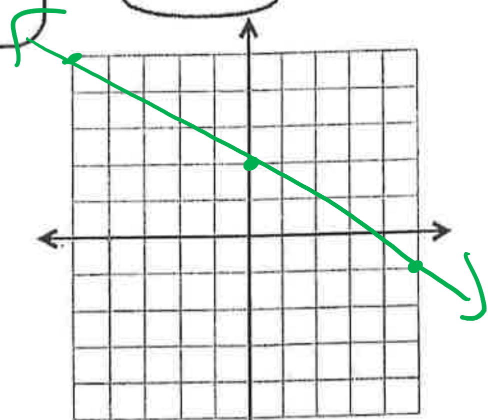
What is the equation of the line in the graph displayed below:



Slope: $\frac{1}{2}$ Y-Intercept: $(0,-2)$

Equation: $y = \frac{1}{2}x - 2$

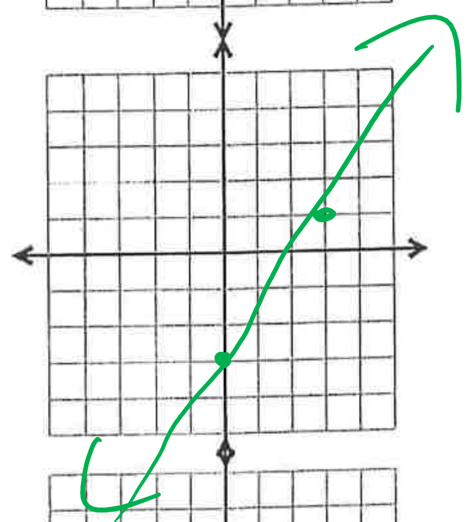
Write the slope as a fraction & y-intercept as a point!



Let's graph the equation

Slope: $-\frac{3}{5}$ Y-Intercept: $(0,2)$

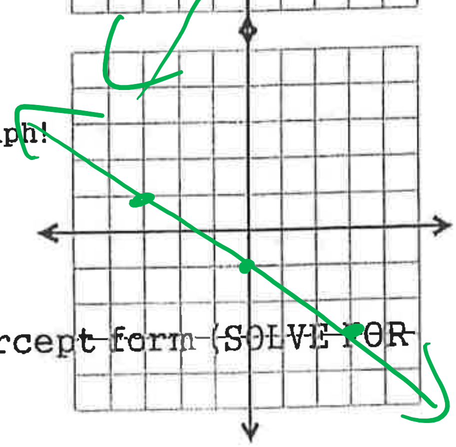
Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$



Let's graph the equation .

Slope: $4/3$ Y-Intercept: $(0,-3)$

Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$



Let's graph the equation .

Remember it MUST be in slope intercept form in order to graph!

Slope: $-2/3$ Y-Intercept: $(0,-1)$

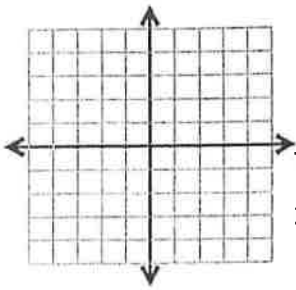
Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$

Let's practice putting equations in slope-intercept form (SOLVE FOR Y!!). Then state the slope and y-intercept.

$3x+2y=9$ $(0, \frac{9}{2})$ $-2x-5y=15$ $3x+\frac{2}{3}y=12$

Slope: $-\frac{3}{2}$ Y-Intercept: $(0, 4.5)$ $y=-\frac{3}{2}x+4.5$	Slope: $-\frac{2}{5}$ Y-Intercept: $(0, -3)$ $y=-\frac{2}{5}x-3$	Slope:____ Y-Intercept:____ $y=-2x+1$

Graph a Vertical line

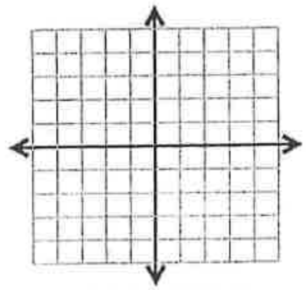


Find 3 points on your line:

WHAT DO YOU SEE?

Equation of your line:

Graph a Horizontal line:



Find 3 points on your line:

WHAT DO YOU SEE?

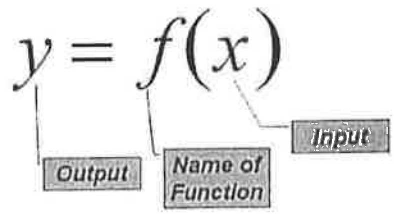
Equation of your line:

Using FUNCTION NOTATION

Output value Input value

$$f(x) = 5x + 3$$

f of *x* equals 5 times *x* plus 3.



Given f(x) = $x^2 - 2$, find:	Given g(x) = 2x + 7, find:	Given h(x) = $-2x^2 + 7x - 11$, find:
f(5)= $(5)^2-2$ $f(5)=23$	g(4)= 15	h(2)= -5
f(−5)= $(-5)^2-2$ $f(-5)=23$	g(−4)= $2(-4)+7$ $-8+7$ -1	h(2a)= $-2(2a)^2+7(2a)-11$ $-8a^2+14a-11$
f(0)= $(0)^2-2$ $f(0)=-2$	g(a)= $2a+7$	3h(−3)= $3 \cdot \boxed{h(-3)}$ $-2(-3)^2+7(-3)-11$ $3 \cdot -50 = -150$

A little more of a challenge: Given f(x) = 2x + 1, find −4[f(3) − f(1)].

-16

$-4(7-3)$

$-4(4)$

-16

Math 3 ~ Unit 1 Day 2 & 3 ~ Solving Systems

A system of equations: A set of 2 or more equations.

A solution is a set of values for the variables that makes all the equations true.

When the "solution" is substituted into the linear equations the result will be a true statement.

3 Ways to solve a system:

1. graph
2. elimination
3. substitution

# of Solutions		**Type of lines**
1	→	intersecting lines
0	→	parallel
infinite	→	same

Method 1: Graphing

- Solve each equation for y.
- Enter the first equation into Y_1.
- Enter the second equation into Y_2.
- Use the INTERSECT option to find where the two graphs intersect (the answer).

  2nd TRACE (CALC) #5 intersect

  Move spider close to the intersection.

  Hit ENTER 3 times.

Let's work an example: $4x - 6y = 12$ $\quad y = \frac{2}{3}x - 2$

$2x + 2y = 6$ $\quad y = -1x + 3$

(3, 0)

Example #2 Application: There are 25 bikes and trikes at the park. The bikes and trikes have 60 wheels in all. How many bikes and trikes are in the park?

b: bikes

t: trikes

$2b + 3t = 60$

$b + t = 25$

Your try. Solve by graphing. (You can do these by hand or with a calculator!)

1. $-3x + 2y = 8$

   $x + 2y = -8$

   $y = \frac{3}{2}x + 4$

   $y = -\frac{1}{2}x - 4$

   (-4, -2)

2. $-2x + 4y = 6$

   $4x - 8y = 12$

   $y = \frac{1}{2}x + \frac{3}{2}$

   $y = \frac{1}{2}x - \frac{3}{2}$

   No solution

3. $2x - y = 3$

   $6x - 3y = 9$

   $y = 2x - 3$

   $y = 2x - 3$

   ∞ Solutions

4. Pedro can choose between two tennis courts at two university campuses to learn how to play tennis. One campus charges $25 per hour. The other campus charges $20 per hour plus a one –time registration fee of $10. Write a system of equations to represent the cost c for h hours of court use at each campus. Find the number of hours for which the costs are the same.

$h=2$

2 hours

$C = 25h$

$C = 20h+10$

Method 2: Algebraically using Elimination

Basic Goal: Add the two equations together so that the x or y is eliminated.

Example #1: $x - 2y = 14$
$x + 3y = 9$

What if the coefficients aren't the same: No Problem! Follow the steps below.

Basic Steps:

1. Arrange equations so variables, equal signs and constants line up vertically.
2. Multiply one or both equations by a value so that one variable in the 1st equation has the opposite coefficient in the other equation.
3. Add the two equations.
4. Solve for the remaining variable.
5. Use the solution from step 4 and substitute into either equation. Solve for the remaining variable.

$X - 2y = 14$
$-1(X + 3y = 9)$

$\cancel{x} - 2y = 14$
$\cancel{-x} - 3y = -9$

$-5y = 5$

$y = -1$ $x = 12$

Example #2: $x - 2y = 12$
$5y = 6x - 23$

$6(x - 2y = 12)$
$-6x + 5y = -23$

$\cancel{6x} - 12y = 72$
$\cancel{-6x} + 5y \quad -23$

$-7y = 49$

$y = -7$

$x = -2$

Practice with Elimination: Solve using elimination

$x-2y=13$ $3x+2y=15$ $4x = 28$ $x=7$ $y=-3$ $(7,-3)$	$2(x-y=5)$ $3x+2y=15$ $2x-2y=10$ $5x = 25$ $x=5$ $y=0$ $(5,0)$	$5(2x+8y=6)$ $2(-5x-20y=-15)$ $10x+40y=30$ $-10x-40y=-30$ $0=0$ ∞ solutions	$3(5x+4y=-14)$ $-2(3x+6y=6)$ $15x+\cancel{12y}=-42$ $-6x-\cancel{12y}=-12$ $9x = -54$ $x=-6 \quad y=4$ $(-6,4)$

Application: The Algebra 2 classes took 60 minutes to answer a combination of 20 multiple-choice and extended-response questions. The class took 2 minutes to answer each multiple choice question and 6 minutes to answer each extended-response question.

a. Write a system of equations to model the relationship between the number of multiple choice questions **m** and the the number of extended-response questions **r**.

b. How many of each type of questions was on the test?

Method 3: Substitution

1. Solve one of the equations for either "x =" or "y =".
   This example solves the second equation for "y=".
2. Replace the "y" value in the first equation by what "y" now equals.
3. Solve this new equation for "x".
4. Place this new "x" value into either of the ORIGINAL equations in order to solve for "y".
5. CHECK the solution in BOTH Equations!

Example #1 :

$$2x-3y=-2$$
$$4x+y=24$$
$$-4x \qquad -4x$$
$$y=-4x+24$$
$$2x-3(-4x+24)=-2$$
$$2x+12x-72=-2$$
$$14x = 70$$
$$x=5$$

$-20+24$

$y=4$

$(5,4)$

Example #2:

$$5x+8y=11$$
$$x+3y=-9$$

Applications with Systems ~ Pick a Method

Suppose that the Greene Cell Phone company charges \$50 per month plus 15 cents per minute while the Johnston Cell Phone Company charges no monthly fee but 25 cents per minute. After how many minutes of phone usage would a monthly phone bill be the same from both companies?

Jake's Surf Shop rents surfboards for \$6.00 plus \$3.00 per hour. Rita's rents them for \$9.00 plus \$2.50 per hour.

- After how many hours of surfing will the rental fee be the same for both surf shops?

- You only want to surf for 2 hours; which Surf Shop should you go to?

Day 3 Graphing Systems of Inequalities

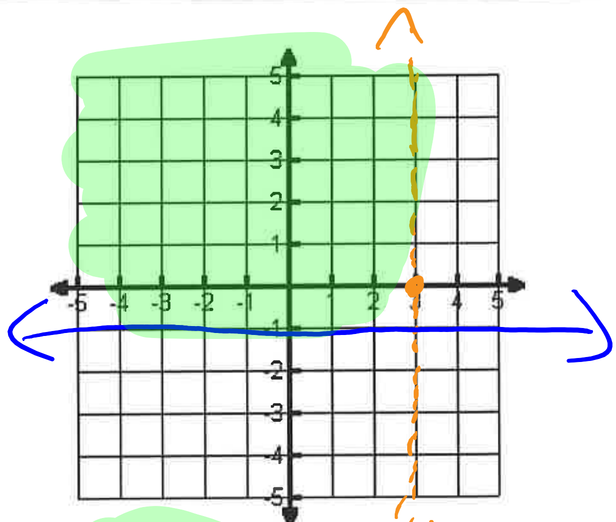
Graphing an Inequality

1. Solve the inequality for y (if necessary). Graph each inequality on the same set of axes.
2. Graph the inequality as if it contained an = sign.
3. Draw the line solid if the inequality is or
4. Draw the line dashed if the inequality is < or >
5. Pick a point not on the line to use as a test point. The point (0,0) is a good test point if it is not on the line.
6. If the point makes the inequality true, shade that side of the line. If the point does not make the inequality true, shade the opposite side of the line.
7. The area where the shading overlaps is the solution to the system of inequalities.

PRACTICE:

Ex: $x < 3$

$y \geq -1$



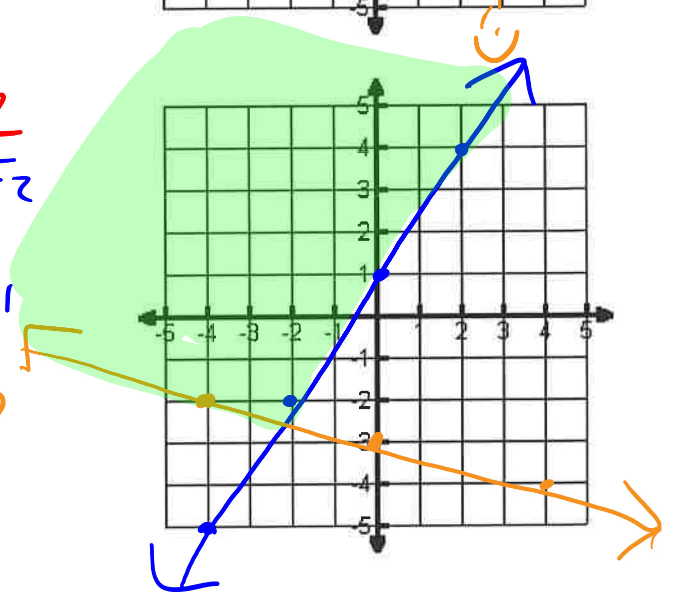
Ex: $3x - 2y \leq -2$

$x + 4y \geq -12$

$$\frac{-2y}{-2} \le \frac{-3x}{-2} \frac{-2}{-2}$$

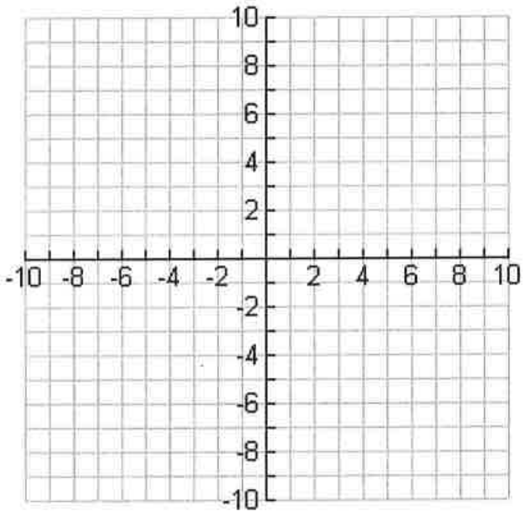
$$y \geq \frac{3}{2}x + 1$$

$$y \geq -\frac{1}{4}x - 3$$

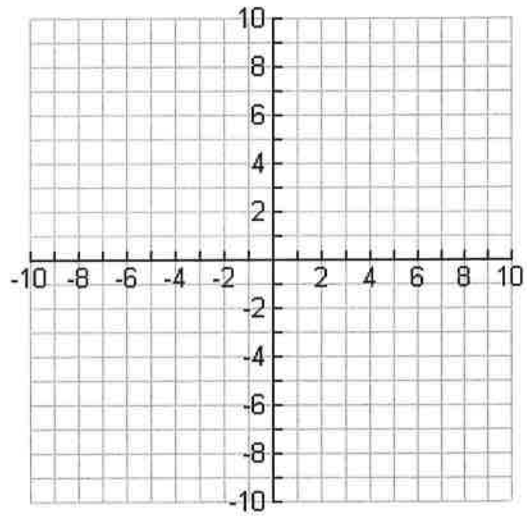


Graph the following inequalities on graph paper.
Used colored pencils to shade.

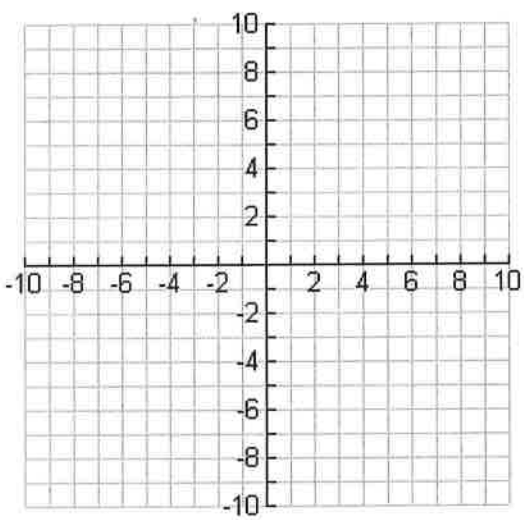
1) $x + 2y \le 8$
$y \le x + 4$



2) $x + y < 3$
$y \ge -x^2$



3) $2x - y \le -3$
$y > 2x^2 + 4x - 4$



4) **Katie works part-time at the Fallbrook Riding Stable. She makes $5 an hour for exercising horses and $10 an hour for cleaning stalls. Because Katie is a full-time student, she cannot work more than 12 hours per week. Graph two inequalities that illustrate how many hours Katie needs to work at each job if she plans to earn not less than $90 per week.**

Write a system of inequalities to model the given scenario.

Use your graphing calculator, to find a possible solution.

Unit 1 Day 4 ~ Absolute Value Functions

Absolute value variable equations are written as:

- $f(x) = |mx + b| + c$
- Graph looks like a right side up or upside down ___________
  - Opens up if the coefficient in front of the absolute value symbols is _____________.

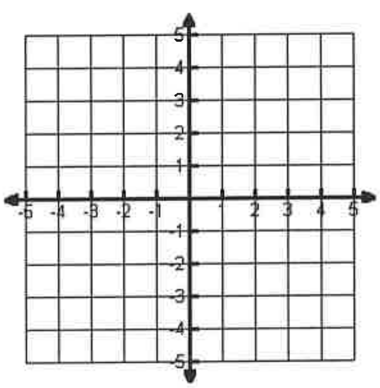
    $f(x) = 4|x + 2| + 3$ opens up

  - Opens down if the coefficient in front of the absolute value symbols is _____________.

    $f(x) = -4|x + 2| + 3$ opens down

- The vertex of the graph will be $\left(-\frac{b}{m}, c\right)$. You can use your calculator to find it!!

Let's start with $f(x) = |x|$ and graph the equation. This is called the parent function.

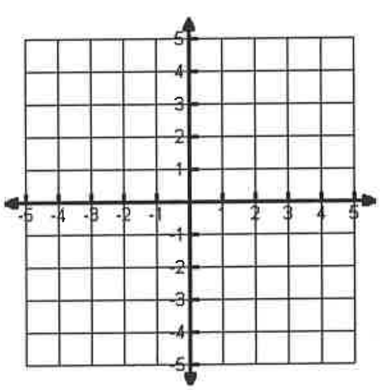


What's the vertex? (__,__)

Does it open up or down? ____

Domain:________________ Range:__________________

You try $f(x) = |x +2|$. How is it different from the parent graph? ___________________



What's the vertex? (__,__)

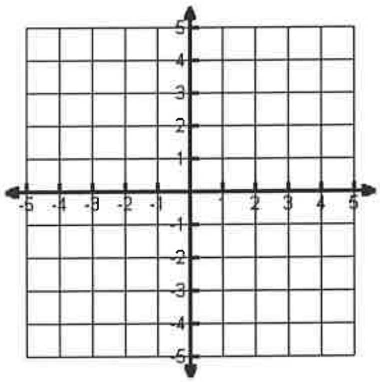
Does it open up or down? ____

Domain:________________ Range:__________________

Now try:

$f(x) = |x| + 2$. How is it different from the parent graph?

What's the vertex? (__,__)



Does it open up or down? ____

Domain:________________ Range:__________________

Vertical Transformations:

A constant added outside the absolute value symbol shifts the graph UP that many units.

$f(x) = |x| + 5$ moves the parent graph ________________

A constant subtracted outside the absolute value symbol shifts the graph DOWN that many units.

$f(x) = |x| - 3$ moves the parent graph ________________

Horizontal Transformations:

A constant added inside the absolute value symbols shifts the graph LEFT horizontally.

$f(x) = |x + 2|$ moves the parent graph ______________________

A constant subtracted inside the absolute value symbols shifts the graph RIGHT horizontally.

$f(x) = |x - 2|$ moves the parent graph ______________________

Reflection over the x-axis:

If you have a ________________ in front of the absolute value, the graph will be reflected, or _______________, over the x-axis.

$f(x) = -|x|$ moves the parent graph ______________________

Vertical Stretch/Compression:

$C \cdot f(x)$, where C is a real number > 0

If $C > 1$, then $f(x)$ has a vertical _________ by a factor of C units.

If $0 < C < 1$, then $f(x)$ has a vertical __________ by a factor of C units.

$f(x) = 2|x|$ How does this compare to the parent?__________________

$f(x) = 0.5|x|$ How does this compare to the parent?__________________

Quick Recap:
In what way would the graph of $y = |x|$ move according to the following equations? Be specific.

1. $y = 4|x+3| - 5$

2. $y = -|x-2| + 7$

Application:

A rainstorm begins as a drizzle, builds up to a heavy rain, and then drops back to a drizzle. The rate r (in inches per hour) at which it rains is given by the function $r = -0.5|t-1| + 0.5$ amd t represents time in hours.

Graph the function.

How long does it rain?

When does it rain the hardest?

What is the rate of the rain after 30 minutes?

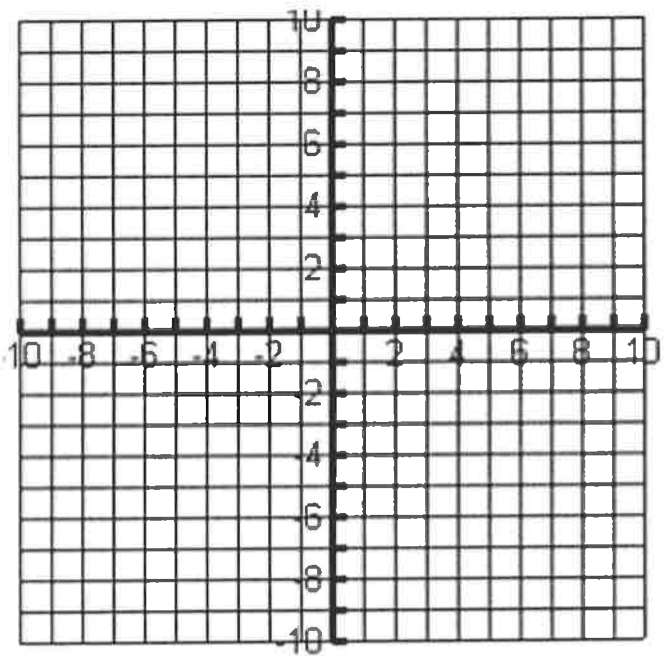
Unit 1: Functions and Their Inverses
Day 5: Introduction to Piecewise Functions Student Notes

Investigation

Graph the following two functions on the same graph.

$f(x) = (x-3)^2 - 4$

$g(x) = 2x - 4$



1. Do the equations overlap?

2. State the Domain and Range.

   f(x) Domain: ________________

   f(x) Range: ________________

   g(x) Domain: ________________

   g(x) Range: ________________

3. Are both equations functions?

4. Find f(0) and g(0).

5. How could we make the two functions continuous? What restrictions needed to be added to our equations?

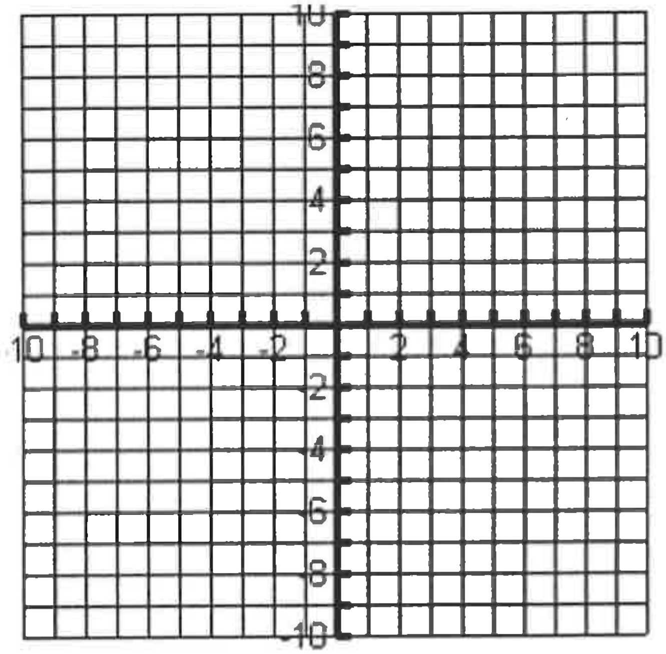
Definition of Piecewise Functions: ____

____

Example 1

Graph the following Piecewise Function. Make sure you restrict your domain for certain "pieces" of the function.

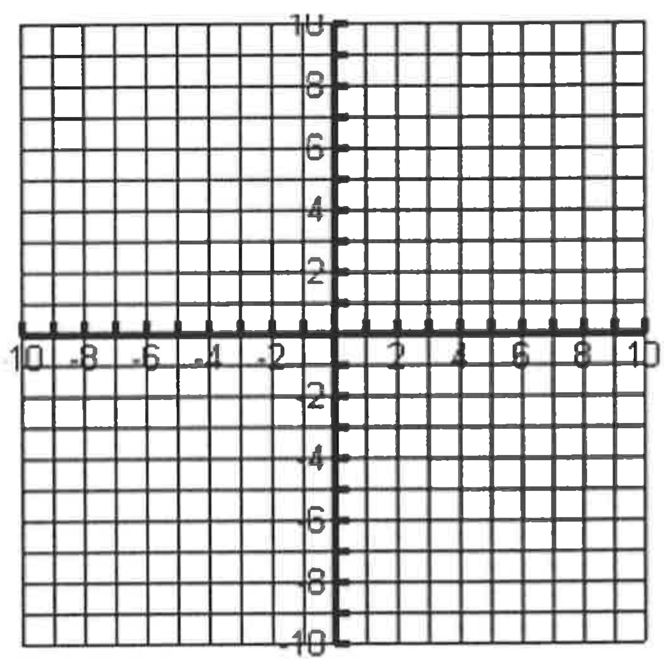
$$f(x)=\begin{cases} x-4, & x<0 \\ -x+4, & x\geq 0 \end{cases}$$



Example 2

Graph the following Piecewise Function. Make sure you restrict your domain for certain "pieces" of the function.

$$f(x)=\begin{cases} x^2, & x\geq -2 \\ x+8, & x<-2 \end{cases}$$

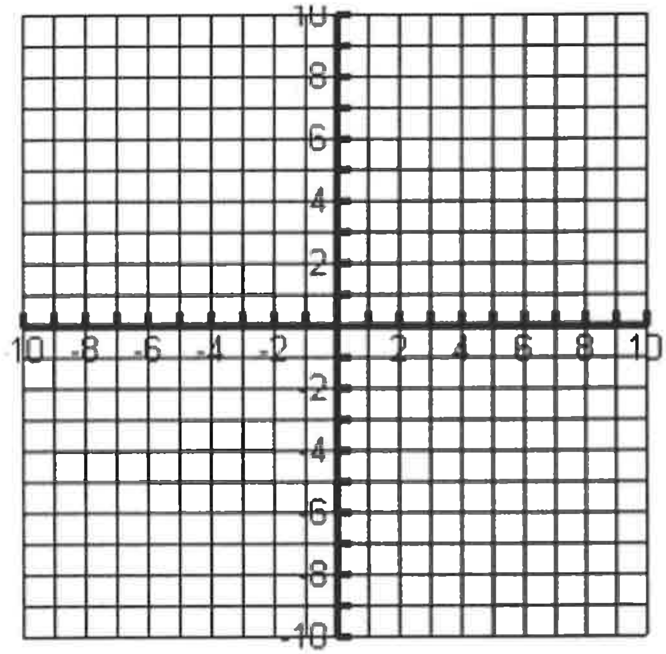


Unit 1: Functions and Their Inverses
Day 6: Graphing and Evaluating Piecewise Functions with Context Student Notes

Example 1

Graph the following Piecewise Function. Make sure you restrict your domain for certain "pieces" of the function.

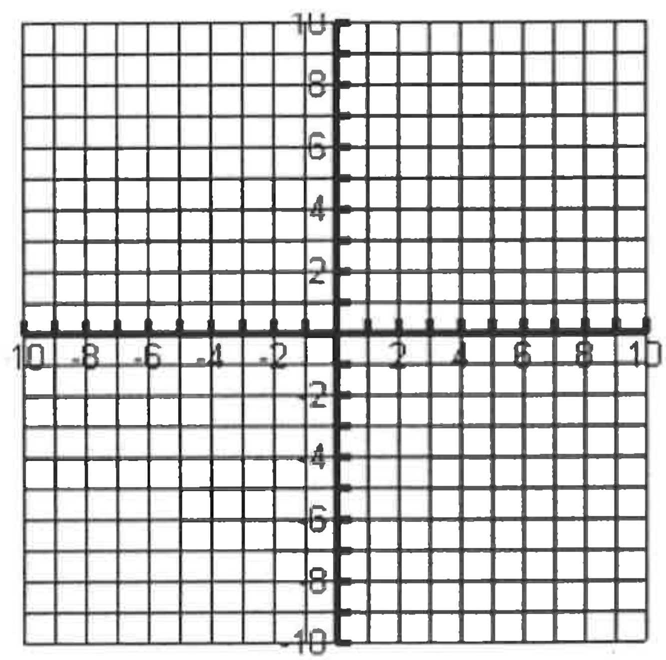
$$f(x)=\begin{cases} -x^2-4, & x<-2 \\ x^2+4, & x\geq -2 \end{cases}$$



Example 2

Graph the following Piecewise Function. Make sure you restrict your domain for certain "pieces" of the function.

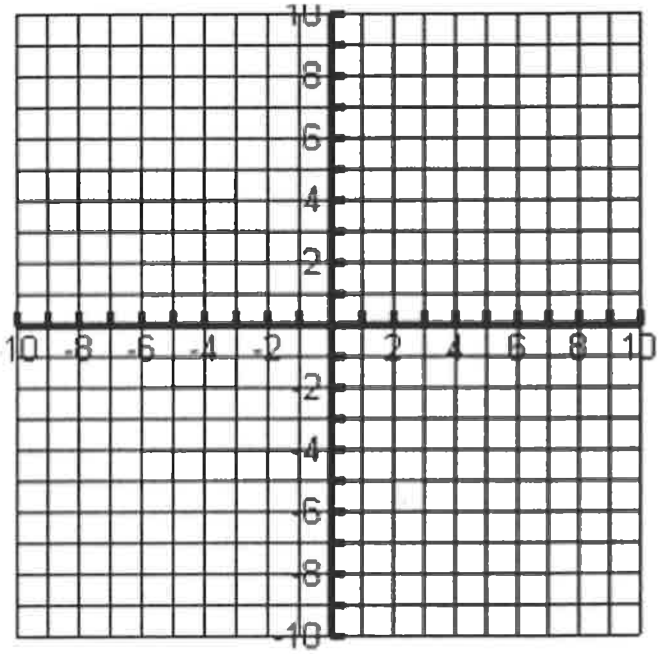
$$f(x)=\begin{cases} x-4, & x\geq -1 \\ \left|x+8\right|, & x<-1 \end{cases}$$



Example 3

Graph the following Piecewise Function. Make sure you restrict your domain for certain "pieces" of the function.

$$f(x)=\begin{cases} -x, & x<-1 \\ 5, & -1\le x\le 1 \\ x, & x>1 \end{cases}$$



Evaluating Piecewise Functions

Given the following piecewise function, evaluate the following.

Hint: You can use your graph from the previous example if needed.

$$f(x)=\begin{cases} -x, & x<-1 \\ 5, & -1\le x\le 1 \\ x, & x>1 \end{cases}$$

f(-9)=	f(-1)=	f(1)=
f(-5)=	f(0)=	f(6)=

Student Try Example

Given the following piecewise function, evaluate the following.

Hint: You can use your graph from the previous example if needed.

$$f(x)=\begin{cases} |x-4|-7, & x\leq -1 \\ 2x-3, & -1<x<1 \\ -x^2-2, & x\geq 1 \end{cases}$$

f(-3)=	f(-1)=	f(1)=
f(-5)=	f(0)=	f(4)=

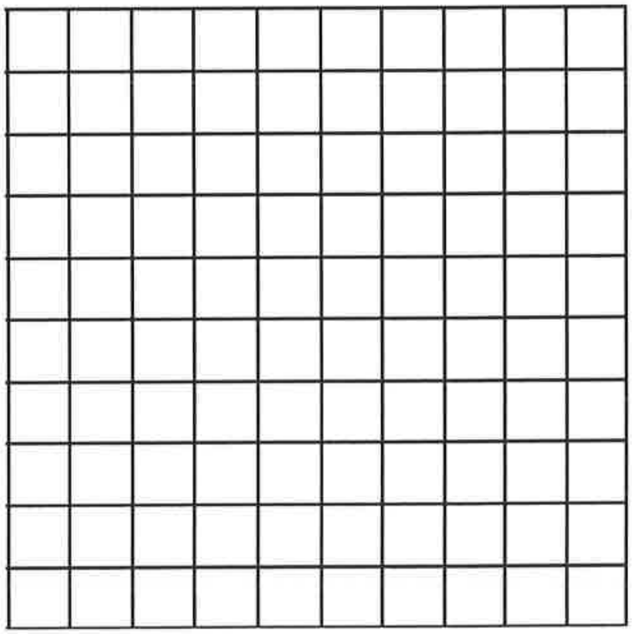
Piecewise Functions in Context

Postal charges for mailing packages depend on both weight and destination and this leads to an application of piecewise functions. For example, the rates for a certain destination are shown in the table below.

Weight in Pounds (x)	**Postage Cost (y)**
Under 1	\$0.80
1 or more, but under 2	\$1.00
2 or more, but under 4	\$1.25
4 or more	\$1.50

Create a piecewise function using the table above.

Graph the piecewise function on the following graph.



Piecewise Functions in Context

There are two parking garages to choose from when parking in Downtown Raleigh.

Parking Garage 1	Parking Garage 2
\$6 Dollars for the First Hour \$5 Dollars for the Second Hour \$4 Dollars for each hour starting on the Third Hour	\$5 Dollars per hour, up to 5 hours \$4 Dollars per hour after that

1. What is the cost to park in both parking garages for 2.75 hours?

2. What is the cost to park in both parking garages for 5.25 hours?

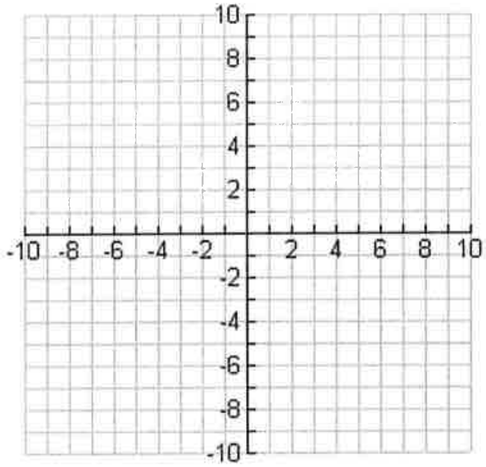
3. What is the best option for parking in Downtown Raleigh?

Day 7 Notes: Finding Inverses

Graph $g(x) = \frac{x}{2} + 4$ and fill in the table. Graph $f(x) = 2x - 8$ and fill in the table:

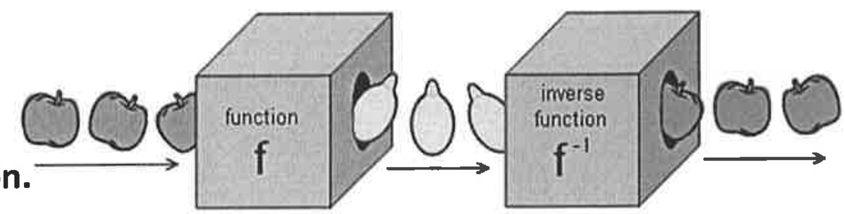
(graph these on the same graph below:)

	g(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

What do you notice about the ordered pairs in each function? Is there a relationship between f(x) and g(x)?



How do I find inverses?

1. Switch _____ and _____ in the equation.

2. Solve for _________.

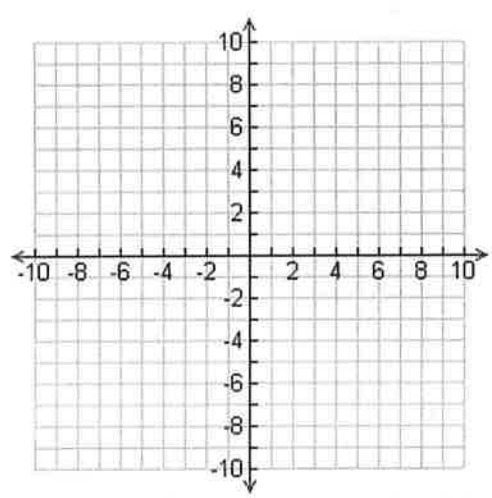
3. Write the answer as _______________.

4. Make the range become the ______ of the new equation, and make the domain become the ______ of the new equation.

The inverse of a relation is a _____________if and only if each ___________ line intersects the graph of the original relation in at most one point. This is the ____________________ _________ Test.

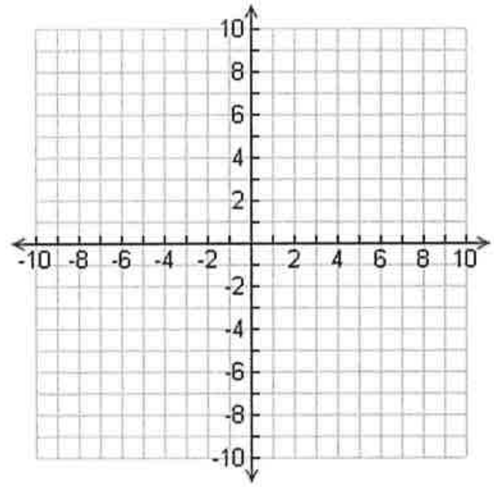
Example 1: $y = \frac{1}{2}x - 7$

Is the inverse a function?_______



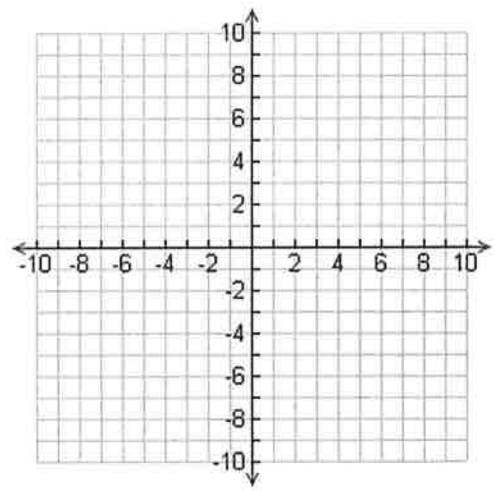
Example 2: $f(x) = 2x - 4$

Is the inverse a function?_______



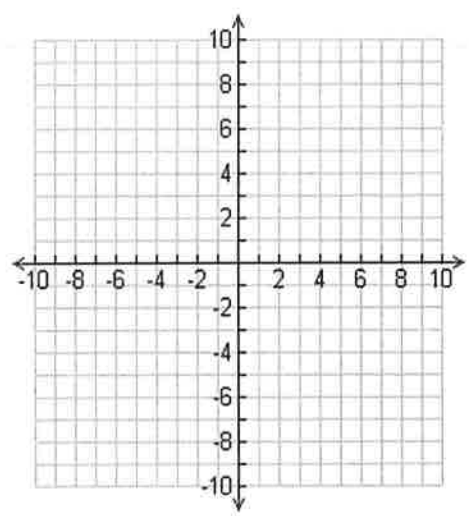
Example 3: $f(x) = (x + 2)^2 - 4$

Is the inverse a function?________



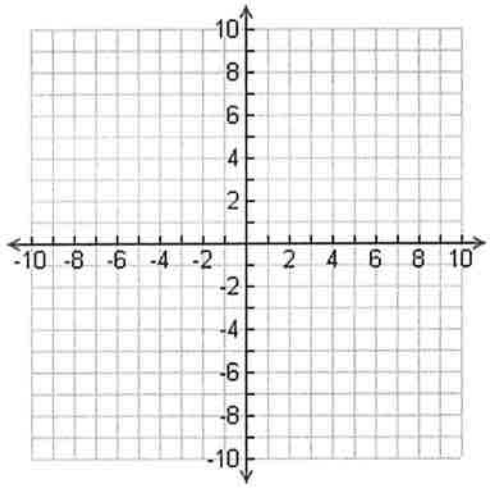
Example 4 $f(x) = 2(x + 7)^3 - 2$

Is the inverse a function?_______



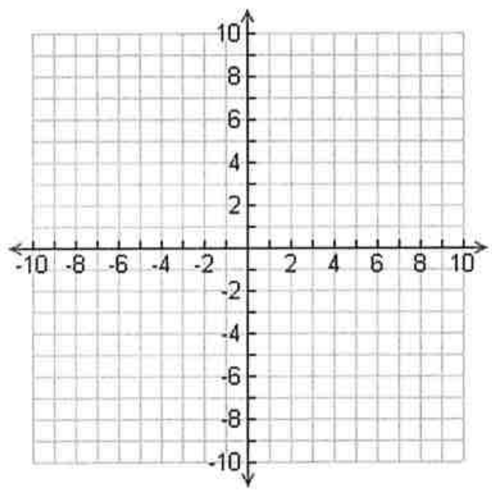
Example 5: $y = \sqrt{x - 3} + 6$

Is the inverse a function?________



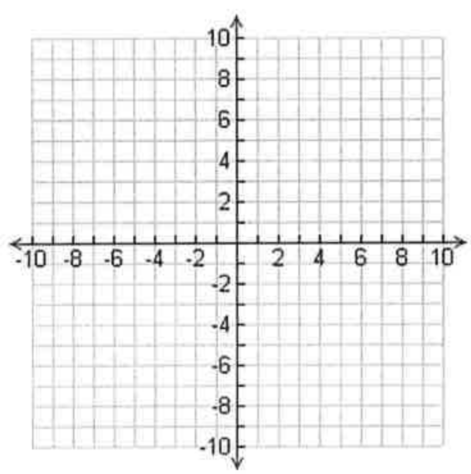
You Try! $y = 4(x + 8)^2 - 5$

Is the inverse a function?________



You Try! $y = \sqrt{x + 7} + 5$

Is the inverse a function?________



Function Operations

Review: Evaluate each function for the given value of x.

Let $f(x) = 3x + 4$. Find $f(-2)$.

Let $g(x) = 2x^2 - 3x + 1$. Find $g(a + 2)$.

Part 1: Basic operations with Functions

Operation	**Definition**	**Examples if f(x)= x + 2 and g(x) = 3x**
Sum	$(f + g)x = f(x) + g(x)$	
Difference	$(f - g)x = f(x) - g(x)$	
Product	$(f \cdot g)x = f(x) \cdot g(x)$	
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$	

Given $f(x) = x^2 - 3x + 1$ and $g(x) = 4x + 5$, find each function.

a) $(f + g)(x)$

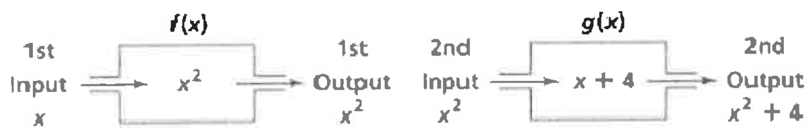
b. $(f - g)(x)$

Given $f(x) = x^2 + 5x - 1$ and $g(x) = 3x - 2$, find each function.

a) $(f \cdot g)(x)$

b. $\left(\frac{f}{g}\right)(x)$

<u>Composition of Functions</u>—taking the output (y-value) of one function and making it the input (x-value) of another function.



<u>Definition of Composition of Functions:</u> The composition of function *f* with function *g* is written $f \circ g(x)$ = f(g(x)).

START ON THE INSIDE & WORK YOUR WAY OUT!!!

Let's Watch a Video: http://www.youtube.com/watch?v=S4AEZElTPDo

$f(x) = x^2 + x$ **$g(x) = 4 - x$**

Find $(f \circ g)(x)$.

Find $(g \circ f)(x)$.

I. Finding a value of a composition given a function

Given $f(x) = x+5$ and $g(x) = x^2 - 2$. Evaluate each expression.

a. $f(g(3))=$

b. $g(f(3)) =$

REMEMBER!!!

$f \circ g(x) = f(g(x))$

NOT

$f(x)g(x)$

Composing is NOT MULTIPLICATION!

II. Finding a composition equation given functions

Given $f(x) = 3x - 2$ and $g(x) = -2x + 4$

Find $f \circ g(x)$

Find $g \circ f(x)$

Function Composition Practice!

1. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ for $f(x) = x + 3$ and $g(x) = x^2 + x - 1$.

2. Evaluate $(f \circ g)(x)$ and $(g \circ f)(x)$ for $x = 2$.

3. Find $(g \circ h)(x)$ and $(h \circ g)(x)$ if $g(x) = 2x$ $h(x) = x^3 + x^2 + x + 1$

4. If $f(x) = x^2 - x$ and $g(x) = x - 1$, what is $f(g(x))$?

5. Tyrone Davis has \$180 deducted from every paycheck for retirement. He can have these deductions taken before taxes are applied, which reduces his taxable income. His federal income tax rate is 18%. If Tyrone earns \$2200 every pay period, find the difference in his net income if he has the retirement deduction taken before taxes or after taxes.