# Math II Unit 4: <br> Variation <br> Lauren Winstead, Heritage High School 

## Direct, Joint, and Inverse Variation

Direct Variation: a relationship between two variables where one is
$\qquad$
$\xrightarrow{\text { Identifying Direct Variation }} \longrightarrow y=k x \quad K=\begin{gathered}\text { constant of variation } \\ \text { Solve for } K\end{gathered}$
Does y vary directly with x ?

| $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: |
| 2 | 8 |
| 3 | 12 |
| 5 | 20 |


| $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: |
| -1 | -2 |
| 3 | 4 |
| 6 | 7 | $\frac{-2}{-1}=2</ x$

$y=4 x$
No
What about in equations? Central question: Can it be rewritten as $y=k x$
Example 1:


You try! Can you get $\mathrm{y}=2 \mathrm{x}+3$ into $\mathrm{y}=\mathrm{kx}$ form?
Make a table of values, and use it to prove your answer.

| $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: |
| 1 | 5 |
| 2 | 7 |
| 3 | 9 |



What one rule have you learned from this example? The yinter kept Must be 0! Example 2: A dripping faucet wastes 1 cup of water if it drips for 3 minutes. The amount of water wasted varies directly with the amount of time passed. Write an equation of direct variation.

You try! The circumference of a circle varies directly with the diameter. What is the constant of variation? Find the diameter of a circle with circumference 105 cm .

$$
105=\pi d
$$

$$
d=\frac{102}{\pi}=33.42 \mathrm{~cm}
$$

Example 3: Write and equation of direct variation that passes through $(9,-1)$.

$$
\begin{aligned}
& y=K x \rightarrow-1=K(a) \rightarrow-1 / a=K \\
& y=-1 / a x
\end{aligned}
$$

You try! Write an equation of direct variation that passes through $(-3,14)$.

Example 4: $y$ varies directly with $x$, and $x=27$ when $y=51$. Find $x$ when $y=11$.

$$
\begin{aligned}
& y=K_{x} \rightarrow \frac{51}{27}=\frac{K .27}{27} \rightarrow K=17 / 9 \rightarrow y=\frac{17}{9} x \\
& \frac{-17}{17 / 4}=\frac{9 x}{11} x_{9} \\
&-9=x
\end{aligned}
$$

You try! y varies directly with x . If $\mathrm{x}=1$ when $\mathrm{y}=5$, find y when $\mathrm{x}=3$.

Inverse Variation: Arelatronship where as one variable increases, the other decreases proportionally such that the product of the variables. Modeling Inverse Variation


Example 5: $x$ and $y$ vary inversely. $x=3$ when $y=-5$. Write the function of inverse variation.

$$
-5=\frac{k}{3} \rightarrow K=-15 \quad y=-15
$$

You try! Decide which type of variation is represented by the data: direct, inverse, or neither.

| $\mathbf{x}$ | 0.5 | 2 | 6 |
| :---: | :---: | :---: | :---: |
| $\mathbf{y}$ | 1.5 | 6 | 18 |


| $\mathbf{x}$ | 0.2 | 0.6 | 1.2 |
| :--- | :--- | :--- | :--- |

$$
\begin{aligned}
& 5 \cdot 1.5=2.25 \\
& 2 \cdot 6=12 \text { Not } \\
& \text { inverse }
\end{aligned}
$$




Joint Variation

| Description | Equation |
| :--- | :--- |
| $y$ varies directly with the square of $x$. | $y=k \cdot x^{2}$ |
| $y$ varies inversely with the cube of $x$. | $y=\frac{k}{x^{3}}$ |
| $z$ varies jointly with $x$ and $y$. | $y=k \cdot x \cdot y$ |
| $z$ varies jointly with $x$ and $y$ and inversely with $w$. | $z=\frac{k \cdot x \cdot y}{w}$ |
| $\frac{z \text { varies directly with } x \text { and inversely with the product of } w \text { and }}{y .}$ | $z=\frac{k \cdot x}{w \cdot y}$ |

Application: The volume of a regular tetrahedron varies directly with the cube of the length of an edge. The volume of a regular tetrahedron with edge length 3 is $\frac{9 \sqrt{2}}{4}$. Find the formula for the volume of a regular tetrahedron.


